

Effective stress

Intelligent
Study Material

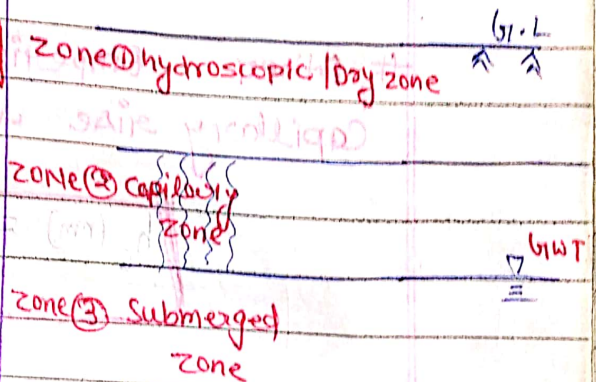
- Effective stress concept was developed by **Terzaghi**
- Effective stress concept applies to a fully saturated soil.

And Related three types of stress

- Total Stress
- Neutral/pore stress
- Effective stress

① **ZONE-1: → Hygroscopic/Dry zone**

↳ The soil in the zone in the hygroscopic or dry condⁿ hence **Bulk or dry unit weight** should be used



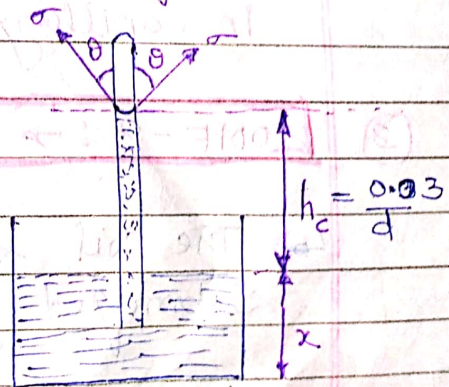
② **ZONE 2: → Capillary zone**

↳ Soil in the zone is under capillary effect generally soil is saturated but may be partial saturated is given in the question due to capillary effect soil is pulled up above the water table

Height of the capillary rise is given as

$$h = \frac{4\sigma \cos\theta}{\rho g d}$$

σ = Surface tension
 d = dia of pipe



$h_c = \frac{0.03}{d}$ (mm)

$d = \text{mm}$
 $h_c = \text{m}$

Capillary water is held above the water table by surface tension

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It has been shown that if we take 'd' as $0.2 D_{10}$, we get good result for capillary rise in sand and silts

for sand and silt

$$h_c = \frac{0.03}{0.2 D_{10}} \quad D_{10} = \text{effective size of the particle in mm.}$$

Other Empirical formula used for capillary rise is as given below.

$$h_c (\text{cm}) = \frac{C}{e D_{10}} \quad e = \text{void ratio}$$

$D_{10} = \text{effective size in cm}$

C = empirical constant = 0.1-0.5 cm²

Capillary rise is function of pore size.

Due to increase in effective stress, shear stress of soil is also increase.

$$\bar{\sigma} \uparrow = \tau \uparrow$$

Due to capillary rise, effective stress in capillary zone increase.

(3) ZONE - 3 : Submerged zone

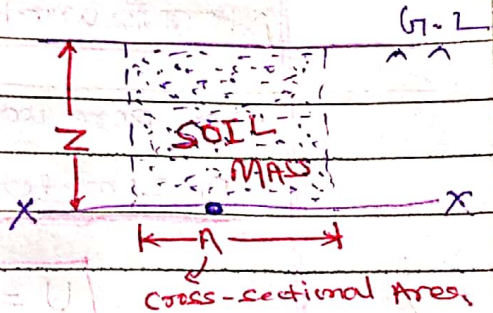
The soil in the zone is under submerged zone.

Total stress: →

↳ It is denoted by σ

↳ Total stress on a plane within a soil mass is force per unit area of soil mass transmitted in normal direction across a plane.

$$\sigma_{xx} = \frac{\text{Wt. of soil}}{\text{cross-sectional area}}$$



$$\sigma_{xx} = \frac{\text{Vol. of soil} \times \gamma_{\text{soil}}}{A}$$

→ Vol. of soil

$$\gamma_{\text{soil}} = \frac{\text{Wt. of soil}}{\text{vol. of soil}}$$

$$\sigma_{xx} = \frac{A \times z \times \gamma_{\text{soil}}}{A}$$

$$\text{Wt. of soil} = \text{Vol. of soil} \times \gamma_{\text{soil}}$$

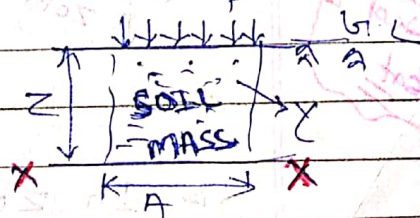
$$\sigma_{xx} = \gamma_{\text{soil}} \times z$$

$$\text{Vol.} = A \times \text{length}$$

- γ_{bulk} = partial saturated
- γ_{dry} = dry condⁿ
- γ_{sat} = saturated condⁿ

↳ It is a stress at any section due to weight of soil and external load above it

$$\sigma_{xx} = \frac{P}{A} + \gamma \cdot z$$



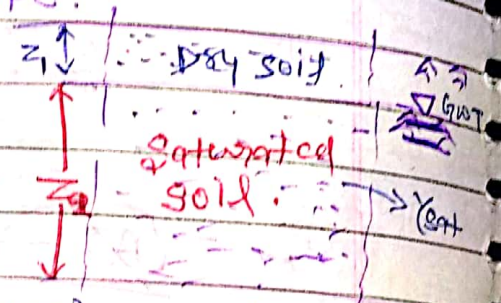
↳ Total stress is a physical parameter which can be measured by suitable arrangement such as by Pressure cell

PORE OR NEUTRAL STRESS (U)

- ↳ It is denoted by U
 - ↳ Static → store is used
 - ↳ Flow
- ↳ It is the pressure of water filling the void space between solid particles.
- ↳ It has no shear component and also measurable quantity
- ↳ Pore water pressure is measured using a piezometer or a stand pipe.

$$U = \gamma_w \times z_g$$

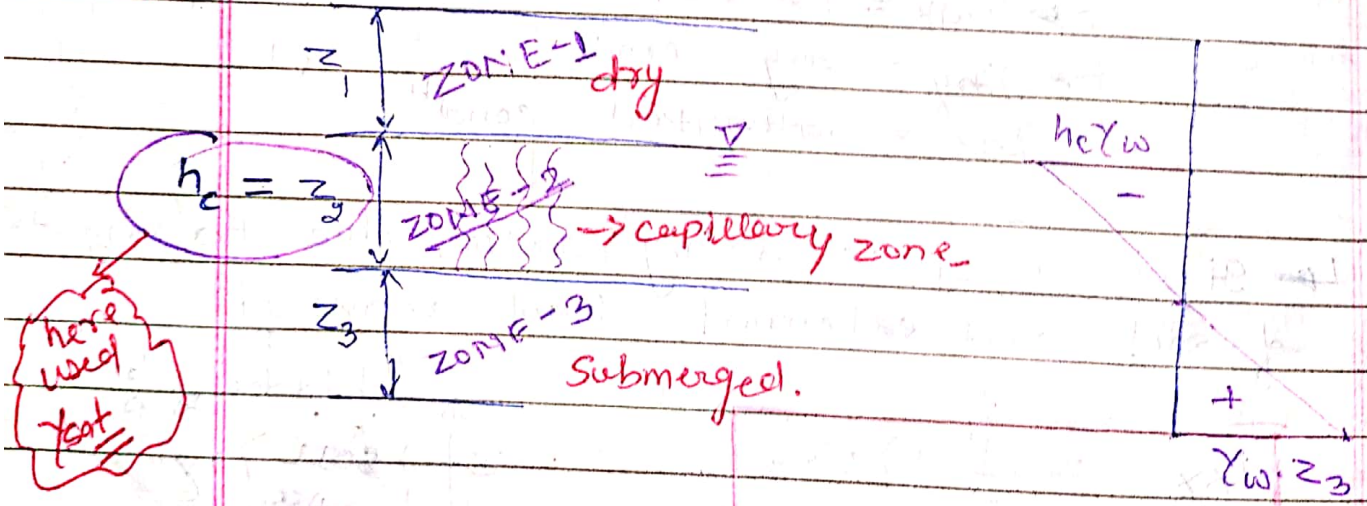
↳ unit wt of water



↳ At Ground water table (GWT) surface pore pressure is zero

↳ It is also a Physical parameter

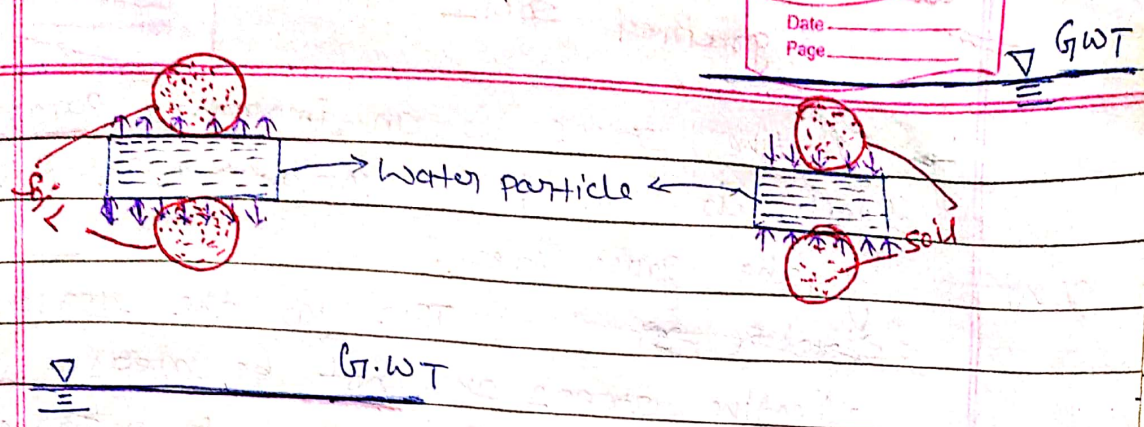
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here used test

↳ In Zone - 1 generally pore water is zero under dry or bulk condⁿ

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Capillary ZONE

Submerged zone

In Capillary zone water is pulled up against gravity hence pore water pressure is in **tension and Negative**

In submerged zone due to hydrostatic pressure of bottom pore water is in **Compression & positive**

$U = -ve$ (tension)

$U = +ve$ (compression)

Effective Stress ! ->

It is denoted by σ

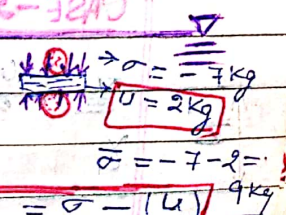
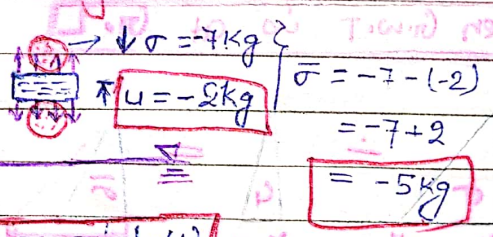
Effective stress is the net grain to grain contact pressure which is transmitted from one layer to another.

Effective stress is defined as equal to the total stress (σ) minus the neutral stress

$\sigma = \sigma - U$

Capillary ZONE

Submerged zone



$\sigma = \sigma - (-U)$
 $= \sigma + U$

$\sigma = \sigma - (U)$
 $= \sigma - U$

$U \rightarrow$ Negative & tension

$U \rightarrow$ positive & compression

also valid for coarse & fine grained soil.

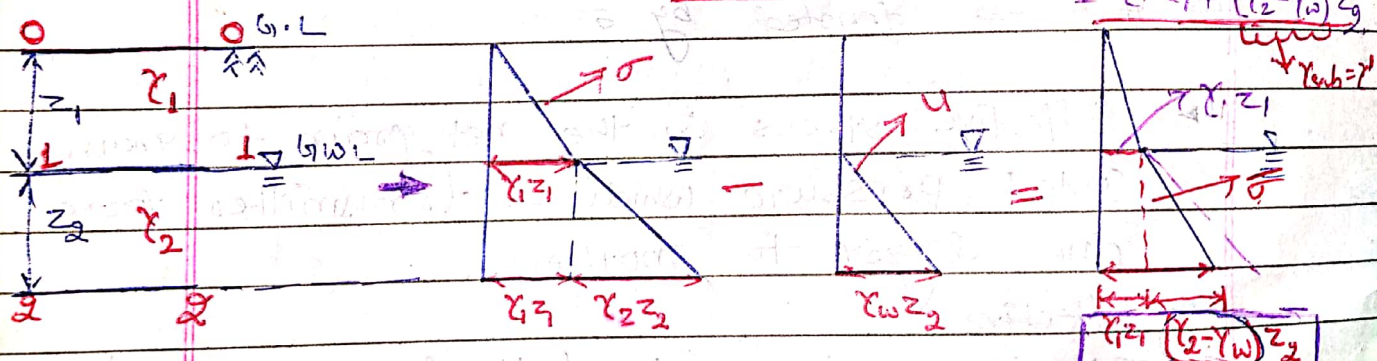
Effective stress is an imaginary parameter which is sum of the contact force divided by the gross area.

~~effective stress~~ This is the reason why effective stress can not be measured hence it is not physical parameter.

NOTE: → Consolidation, settlement, shear strength, bearing capacity are the function of effective stress and not total stress.

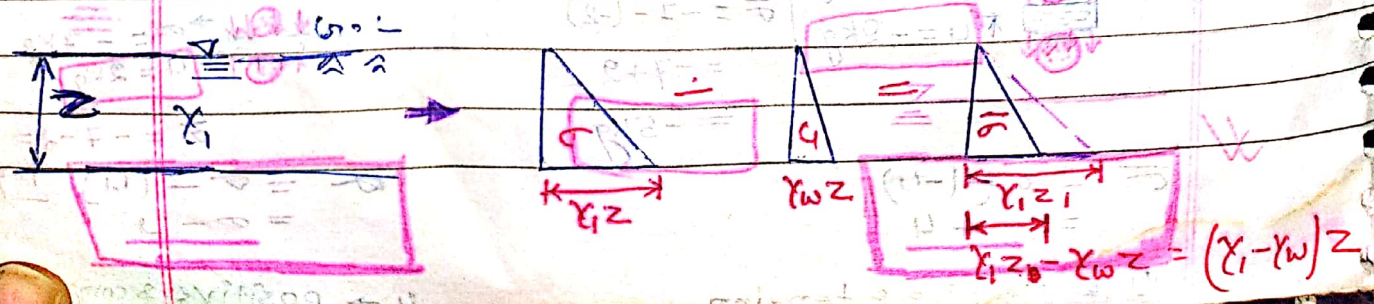
CASE - 1: → GWT at a depth z_1 from G.L.

$\sigma_{00} = 0$ $u_{00} = 0$ $\sigma_{00} = 0$
 $\sigma_{1-1} = \gamma_1 z_1$ $u_{1-1} = 0$ $\sigma_{1-1} = \gamma_1 z_1$
 $\sigma_{2-2} = \gamma_1 z_1 + \gamma_2 z_2$ $u_{2-2} = \gamma_w z_2$ $\sigma_{2-2} = \gamma_1 z_1 + \gamma_2 z_2 - \gamma_w z_2$
 $\sigma_{2-2} = \gamma_1 z_1 + (\gamma_2 - \gamma_w) z_2$



$\gamma' = \gamma_{sub} = \gamma_{sat} - \gamma_w = \text{Submerged unit wt.}$
 $\gamma_2 - \gamma_w$

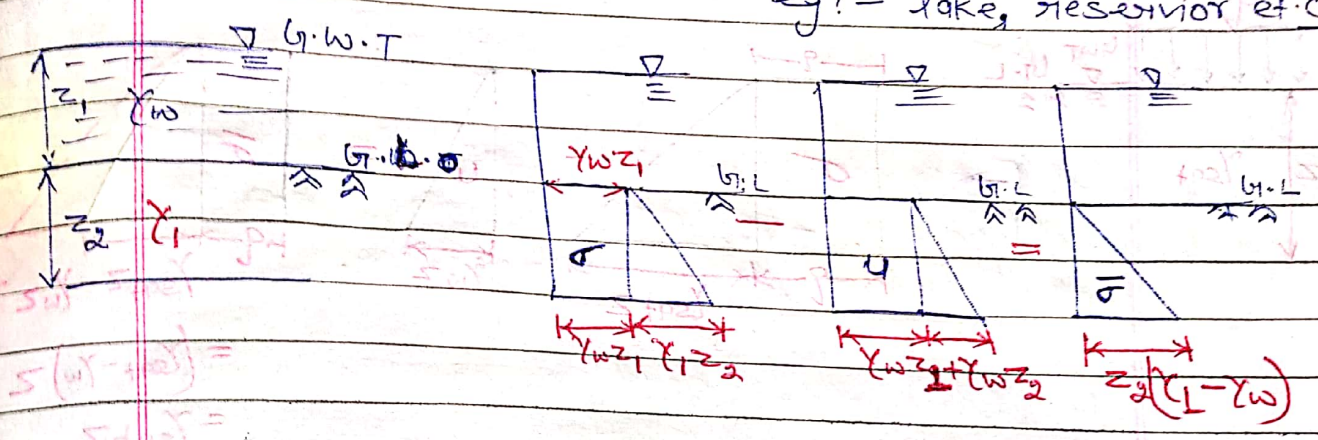
CASE - 2: → When GWT is at G.L.



SOIL and water is incompressible

CASE → 3 :-> When G.W.T is above G.L

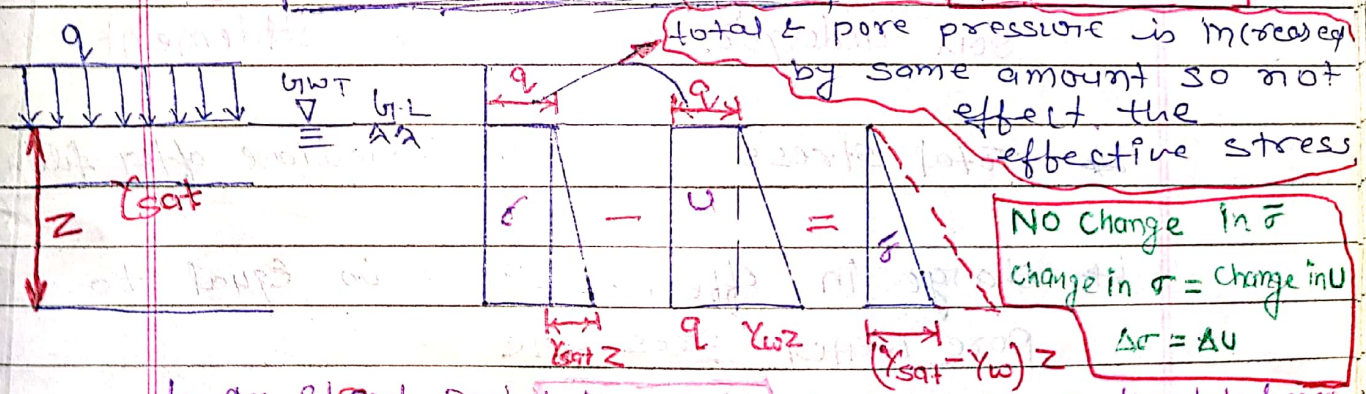
eg! - lake, reservoir etc



NOTE → So we can say when w.T rises above G.L then effective stress below the G.L remain unaffected.

CASE - 4 :-> Effect of surcharge Applying

Ⓐ **SHORT TERM EFFECT** :-> G.W.T at G.L



↳ In short period due to applied surcharge the total pres. and pore press. increased by some amount of $\frac{1}{2}$ of surcharge hence there is no change in effective stress.

Total pore water press. = static pore water press. + Excess pore water pressure

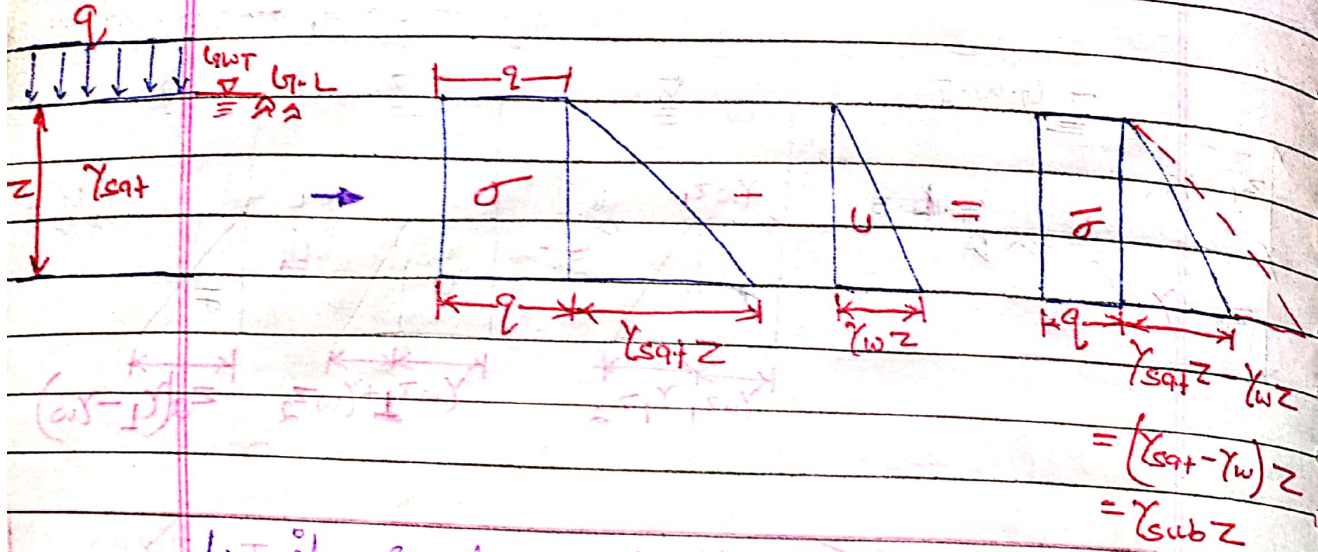
Excess pore water pressure = load

develop due to sudden application of load.
 $load = E \cdot f \cdot \omega \cdot P$

Only one case of capillary action where effective stress is greater than the total stress



(B) LONG TERM EFFECT: → GWT at G.L



→ If surcharge continues to act for long time, then this excess pore water pressure (q) reduces to zero by following the pore water pressure into surrounding low pore water pressure area.

→ Due to this the volume of void below surcharge reduces and soil undergoes consolidation settlement.

→ Total stress same as immediate after filling

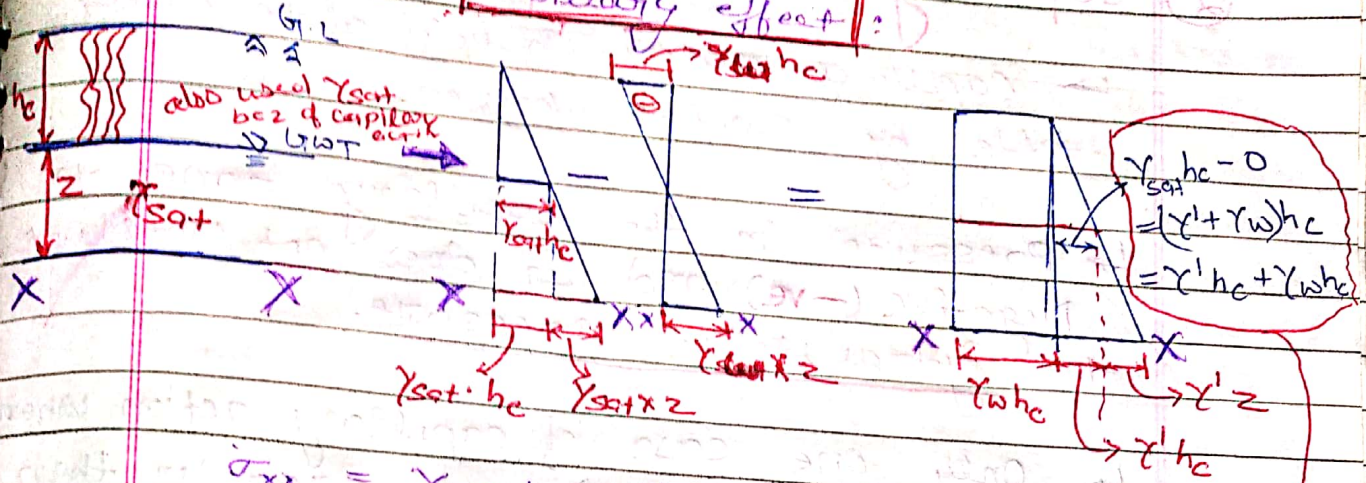
→ change in effective stress is equal to pore water pressure

$$\Delta \sigma' = \Delta u$$

→ Reduction of excess pore water pressure as drainage takes place is called **Dissipation**.

Dissipation is complete, soil is said to be **Drained Condition**

CASE - 5 :- Capillary effect :-



$$\sigma_{xx} = \gamma_{sat} \cdot hc + \gamma_{sat} \cdot z$$

$$u_{xx} = \gamma_w z$$

$$\bar{\sigma}_{xx} = (\gamma_{sat} \cdot hc + \gamma_{sat} \cdot z) - (\gamma_w z)$$

$$= \gamma_{sat} \cdot hc + \gamma_{sat} \cdot z - \gamma_w z$$

$$= \gamma_{sat} \cdot hc + (\gamma_{sat} - \gamma_w) z$$

$$= \gamma_{sat} \cdot hc + \gamma' z$$

$$= \gamma' hc + \gamma_w hc + \gamma' z$$

$$= \gamma' hc + \gamma' z + \gamma_w hc$$

$$\begin{aligned} \gamma_{sat} \cdot hc - 0 &= \gamma_{sat} \cdot hc \\ &= (\gamma_w + \gamma') hc \\ &= \gamma_w hc + \gamma' hc \end{aligned}$$

Capillary in soil.

(Groundwater can exist in two forms)

① Phreatic or Gravitational water

② Capillary water

↳ Subjected to gravitation force & saturated the void

↳ generally called GWT completely

↳ At GWT surface pressure is

atmospheric pressure and below which pressure is hydrostatic

② Capillary water: -

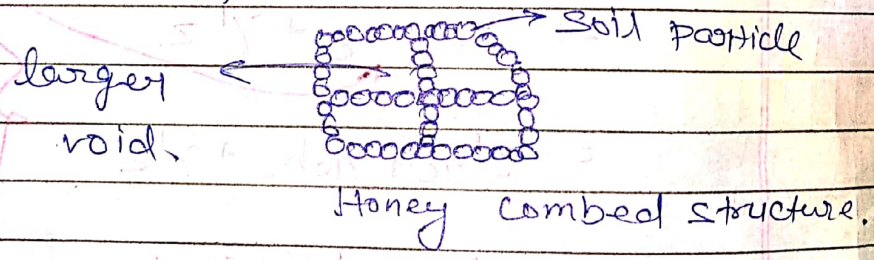
↳ Capillary water is held above water table by surface tension.

↳ Pressure in the capillary zone is Negative (-ve) and it does not contribute to hydraulic pressure.

↳ Only one case of capillary action where effective stress ($\bar{\sigma}$) is greater than total stress (σ)

↳ **Bulking of sand** also occurs due to **capillarity**.

capillarity produces apparent cohesion which holds the particles in clusters, enclosing honey combs

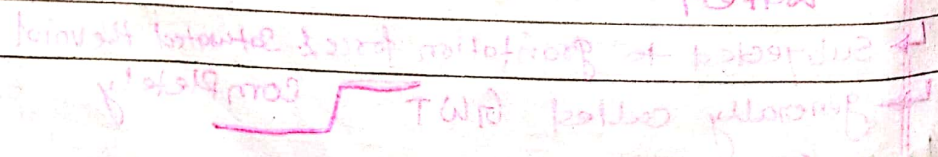


$\sigma = \gamma \cdot h$
 $\bar{\sigma} = \gamma \cdot h - \gamma_w \cdot h_c$
 $\bar{\sigma} = \gamma \cdot h - \gamma_w \cdot \frac{2T}{r}$
 $\bar{\sigma} = \gamma \cdot h - \gamma_w \cdot \frac{2 \cdot 73 \times 10^{-8}}{0.0001}$

Capillary water

③

① Phreatic or gravitational water



① Q → A 10m thick clay layer is underlain by sand layer of 20m depth. The GWL is 5m below surface of clay layer. The soil above GWL is capillary saturated of now water table rises to surface. Then the effective stress at point P on the interface of two soils will

- Ⓐ increase by $5\gamma_w$
- Ⓑ remain unchanged
- Ⓒ decrease by $5\gamma_w$
- Ⓓ decrease by $10\gamma_w$

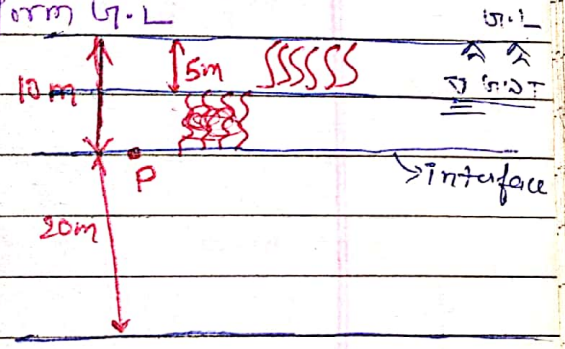
initial condition → GWL at 5m from G.L

$$\sigma_{p_i} = \gamma_{sat} \times 10$$

$$u_{p_i} = -(5\gamma_w)$$

$$\bar{\sigma}_{p_i} = \gamma_{sat} \times 10 - (-5\gamma_w)$$

$$= 10\gamma_{sat} + 5\gamma_w$$



final condition: → GWL is rises to surface of G.L

$$\sigma_{p_f} = \gamma_{sat} \times 10$$

$$u_{p_f} = -\gamma_w \times 10$$

$$\bar{\sigma}_{p_f} = 10\gamma_{sat} - (10\gamma_w)$$

Change = $\bar{\sigma}_{p_f} - \bar{\sigma}_{p_i}$

$$= (10\gamma_{sat} - 10\gamma_w) - (10\gamma_{sat} + 5\gamma_w)$$

$$= 10\gamma_{sat} - 10\gamma_w - 10\gamma_{sat} - 5\gamma_w$$

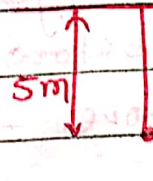
$$= -15\gamma_w$$

Q. The ground condⁿ at a sight are given below

$$w = 20\% = 0.20$$

$$G = 2.7$$

$$\gamma_w = 10 \text{ kN/m}^3$$



$$w = 20\%$$

$$G = 2.7$$

$$\gamma_w = 10 \text{ kN/m}^3$$

Q-1 $\gamma_{sat} = ?$

Q-2 $\sigma, U, \& \bar{\sigma}$ at P

$$\text{① } \gamma_{sat} = \frac{(G+e)\gamma_w}{1+e}$$

$$S \cdot e = wG$$

$$= \frac{(G + wG)\gamma_w}{1 + wG}$$

$$= \frac{(1+w)G\gamma_w}{1+wG}$$

$$= \frac{(1+0.20)10 \times 2.7}{1+2.7 \times 0.20}$$

$$= 2.7 \times 7.79 \text{ kN/m}^3$$

$$= 21.033 \text{ kN/m}^3$$

$$\text{② } \sigma_p = \gamma_{sat} \times 5 \text{ m}$$

$$= 21.033 \times 5$$

$$= 105.16 \text{ kN/m}^2$$

$$U_p = \gamma_w \times 5 \text{ m}$$

$$= 10 \times 5$$

$$= 50 \text{ kN/m}^2$$

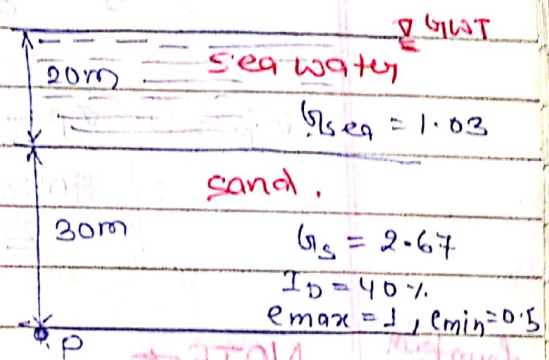
$$\bar{\sigma}_p = \sigma_p - U_p$$

$$= 105.16 - 50 = 55.16 \text{ kN/m}^2$$

3 Q → A sand layer found at sea floor under 20m water table has relative density 40%.
 $e_{max} = 1$, $e_{min} = 0.5$, $G_s = 2.67$
 Assuming Sp. gravity of sea water = 1.03
 $\gamma_w = 9.81 \text{ kN/m}^3$ for fresh water

- ① What would be $\bar{\sigma}$ in kpa at 30m depth into the sand layer
- ② What would be change in $\bar{\sigma}$ at 30m depth into the sand layer if sea water level rises permanently by 2m.

$I_D = 40\%$
 $e_{max} = 1$, $e_{min} = 0.5$
 $G_s = 2.67$
 $G_{sea} = 1.03$
 $\gamma_w = 9.81 \text{ kN/m}^3$



$$I_D = \frac{e_{max} - e}{e_{max} - e_{min}}$$

$$0.40 = \frac{1 - e}{1 - 0.5}$$

$$0.40 \times 0.5 = 1 - e$$

$$e = 1 - 0.2 = 0.8$$

$$\gamma_{sat} = \frac{(G_s + e) \gamma_{sea}}{1 + e}$$

$$= \frac{(2.67 + 0.8) \times 10.104}{1 + 0.8}$$

$$\gamma_{sat} = 19.47 \text{ kN/m}^3$$

bcz here sea water is fill in the pores of soil so we require γ_{sea} .

$$G_{sea} = \gamma_{sea} / \gamma_w$$

$$\gamma_{sea} = G_{sea} \cdot \gamma_w$$

$$= 1.03 \times 9.81$$

$$\gamma_{sea} = 10.104$$

$$\textcircled{1} \sigma_p = 20 \times \gamma_{sea} + \gamma_{sat} \times 30$$

$$= 20 \times 10.104 + 19.47 \times 30$$

$$= 775.8 \text{ kN/m}^2$$

$$U_0 = 50 \times \gamma_{sea}$$

$$= 50 \times 10.104$$

$$= 505.2$$

$$\bar{\sigma}_p = \sigma_p - U_p$$

$$= 775.8 - 505.2 = 270.6$$

$$\bar{\sigma}_p = \sigma_p - U_p$$

$$= 786.18 - 505.2$$

$$= 280.98 \text{ KN/m}^2$$

② When water table permanently rise 2m.

$$\sigma_p = 2 \times \gamma_{\text{sea}} + 20 \times \gamma_{\text{sea}} + 30 \times \gamma_{\text{sat}}$$

$$= 22 \times \gamma_{\text{sea}} + 30 \times \gamma_{\text{sat}}$$

$$= 22 \times 10.104 + 30 \times 19.4$$

$$= 804.288 \text{ KN/m}^2$$

$$U_p = 2 \times \gamma_{\text{sea}} + 20 \times \gamma_{\text{sea}} + 30 \times \gamma_{\text{sea}}$$

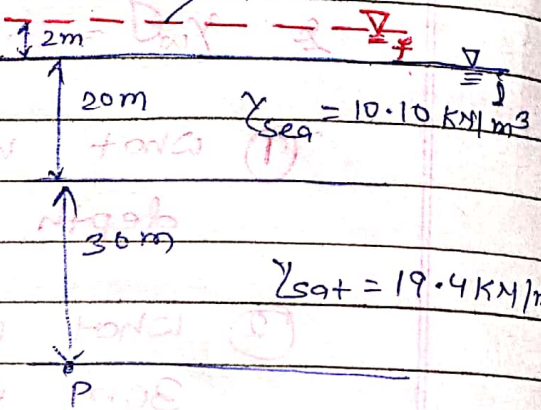
$$= 52 \times \gamma_{\text{sea}} = 52 \times 10.104$$

$$= 525.408$$

$$\bar{\sigma}_p = \sigma_p - U_p$$

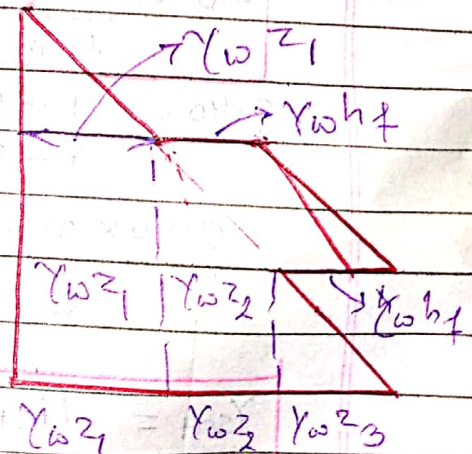
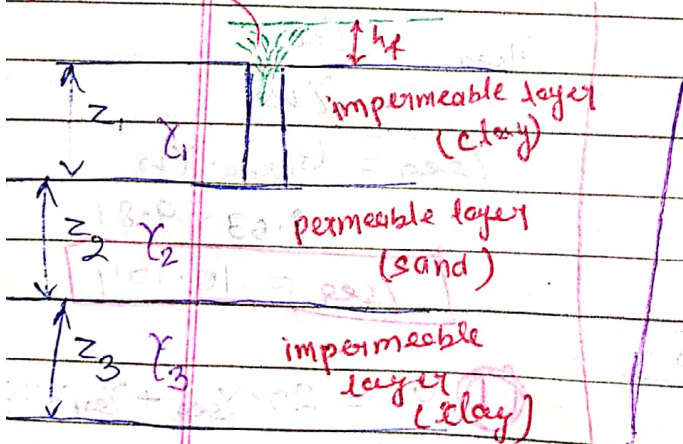
$$= 804.288 - 525.408$$

$$= 278.88 \text{ KN/m}^2$$



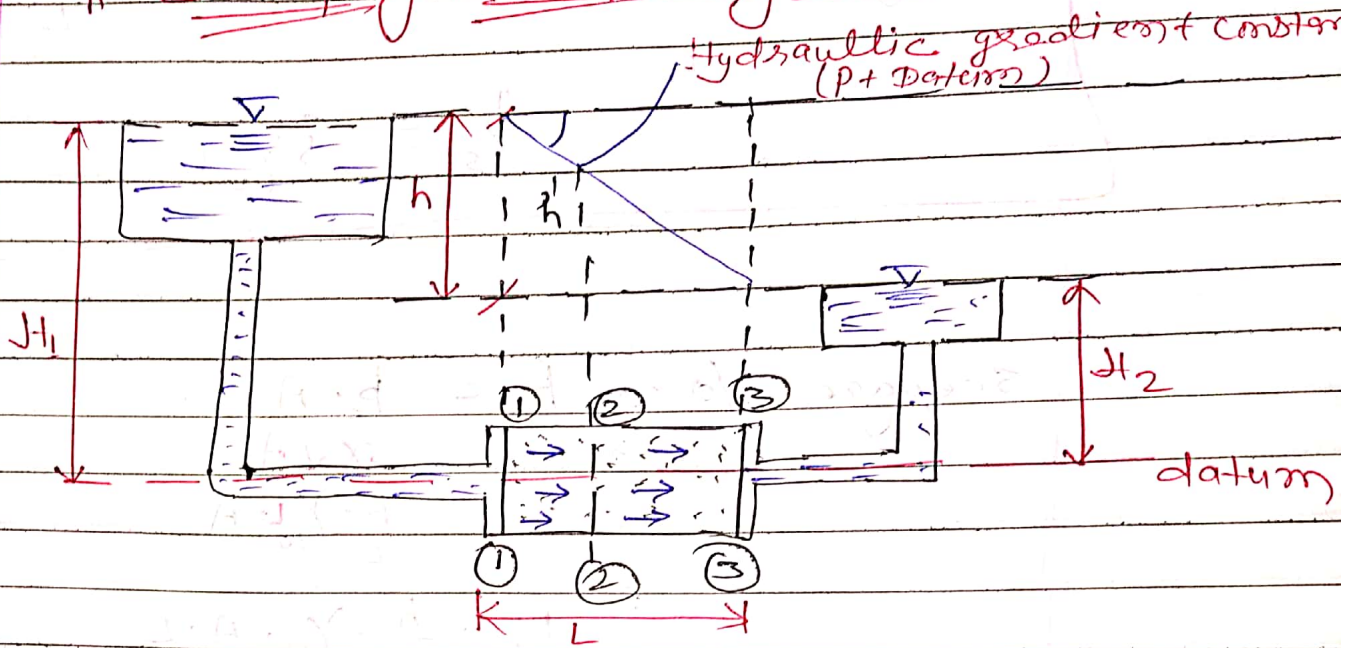
fountain

NOTE →



$$\gamma_w z_1 = \gamma_w z_2 = \gamma_w z_3$$

* Seepage Analysis *



→ Seepage force is a drag force exerted by on the soil molecules which always acts in the direction of flow. It is due to viscous friction of water on solids and is directly proportional to the hydraulic head of water at that point.

$$\text{Seepage pressure at } \textcircled{1} - \textcircled{2} = h' \gamma_w$$

$$\text{" " at } \textcircled{2} - \textcircled{2} = h' \gamma_w$$

$$\text{" " at } \textcircled{3} - \textcircled{3} = 0$$

→ Seepage pressure $\equiv (p_s) = h \gamma_w = \frac{h}{L} \times \gamma_w \cdot L$

$$p_s = i \gamma_w L$$

Seepage force $F_s = p_s \cdot A$
 $= i \gamma_w L A$
 $= \frac{h \cdot \gamma_w \cdot L \cdot A}{L}$

$$F_s = \frac{h}{L} \cdot \gamma_w \cdot A \cdot L$$

$$F_s = i \gamma_w \cdot V$$

Seepage force/unit volume $= i \gamma_w$

* Special Case :-

If the seepage flow in vertical direction then it will increase or decrease the vertical effective stress (grain to grain contact pressure)

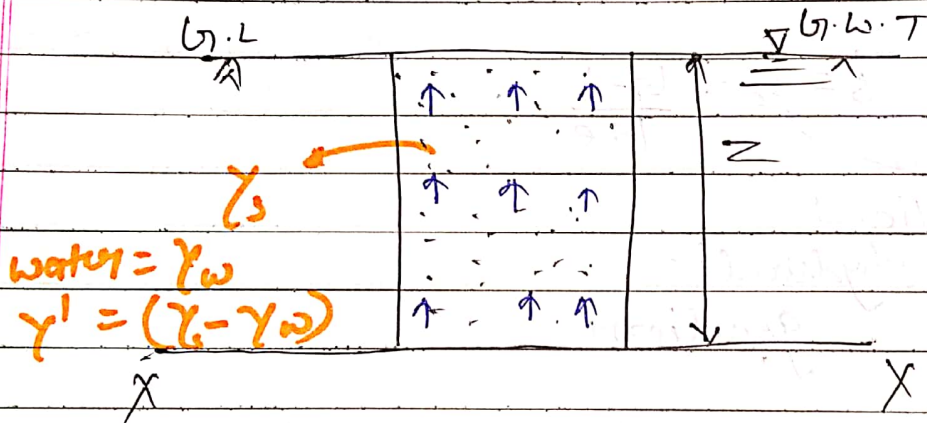
$$\text{Net effective vertical pressure} = \bar{\sigma} \pm p_s$$

If flow in down ward

$$\bar{\sigma}_{net} = \bar{\sigma} + p_s$$

If flow in upward

$$\bar{\sigma}_{net} = \bar{\sigma} - p_s$$



If hydraulic head of water available at section $x-x = h$

$$p_{s(x-x)} = s z \gamma_w = h \gamma_w$$

$$\bar{\sigma}_{net} = \bar{\sigma}_{ax} - p_s$$

$$= \gamma' z - s z \gamma_w$$

If net effective stress becomes zero
 $\bar{\sigma}_{net} = 0$

$$i \gamma_w = \gamma' z$$

$$i = \frac{\gamma'}{\gamma_w} z$$

$$i_c = \frac{(\gamma' - 1) \gamma_w}{\gamma_w}$$

$$i = i_c = \frac{\gamma' - 1}{1 + e}$$

Critical
hydraulic
gradient

→ In cohesionless soil such as sand and fine gravels if net effective stress ($\bar{\sigma}_{net}$) b/w particle reduced becomes zero due to upward seepage force, then in such condition the particle will start floating and may flow with water. Such condition is called quick sand condition.

FOR QUICK SAND CONDITION: →

Where net grain to grain contact pressure becomes zero

→ And such condition occurs at hydraulic gradient or above it called critical hydraulic gradient

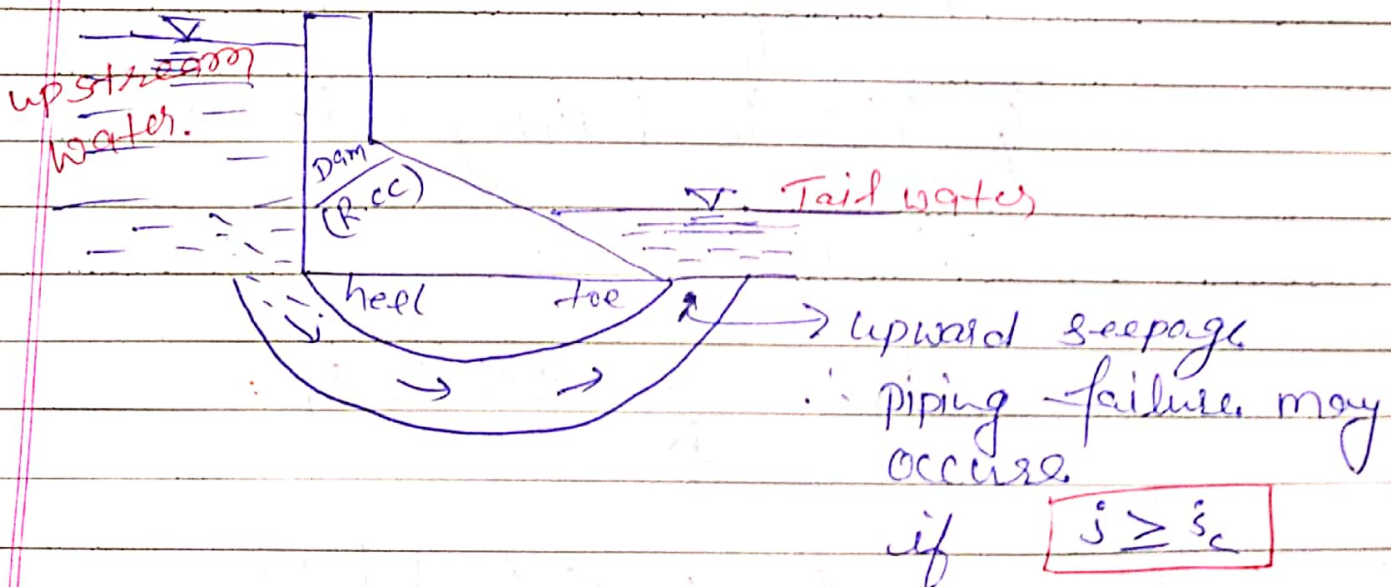
$$G_c = 2.6 \text{ to } 2.7$$

$$i_c = \frac{G_c \cdot \gamma}{1 + e}$$

$$e = 0.6 \text{ to } 0.7$$

$$\frac{2.6 - 1}{1 + 0.6} = \frac{1.6}{1.6} = 1$$

Quick Sand condition may result in piping failure below hydraulic structure near the toe.



F.O.S against piping

$$F_c = \frac{i_c}{i}$$

*** Flow Net ***

→ It is a graphical representation of energy flow through the flow filled It is graphical solution of Laplace equation.

For 2D

(a) For isotropic soil :-

$$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} = 0 \quad \left. \begin{array}{l} \nabla^2 \phi = 0, H = \text{initial head} \\ \therefore \text{isotropic} \end{array} \right\} \text{Square flow net}$$

(b) For non-isotropic soil

$$k_x \frac{\partial^2 H}{\partial x^2} + k_y \frac{\partial^2 H}{\partial y^2} = 0 \quad \text{distorted flow Net}$$

$$\frac{k_x}{k_y} \frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} = 0$$

$$x = x_T \sqrt{\frac{k_x}{k_y}}$$

→ scale of flow net.

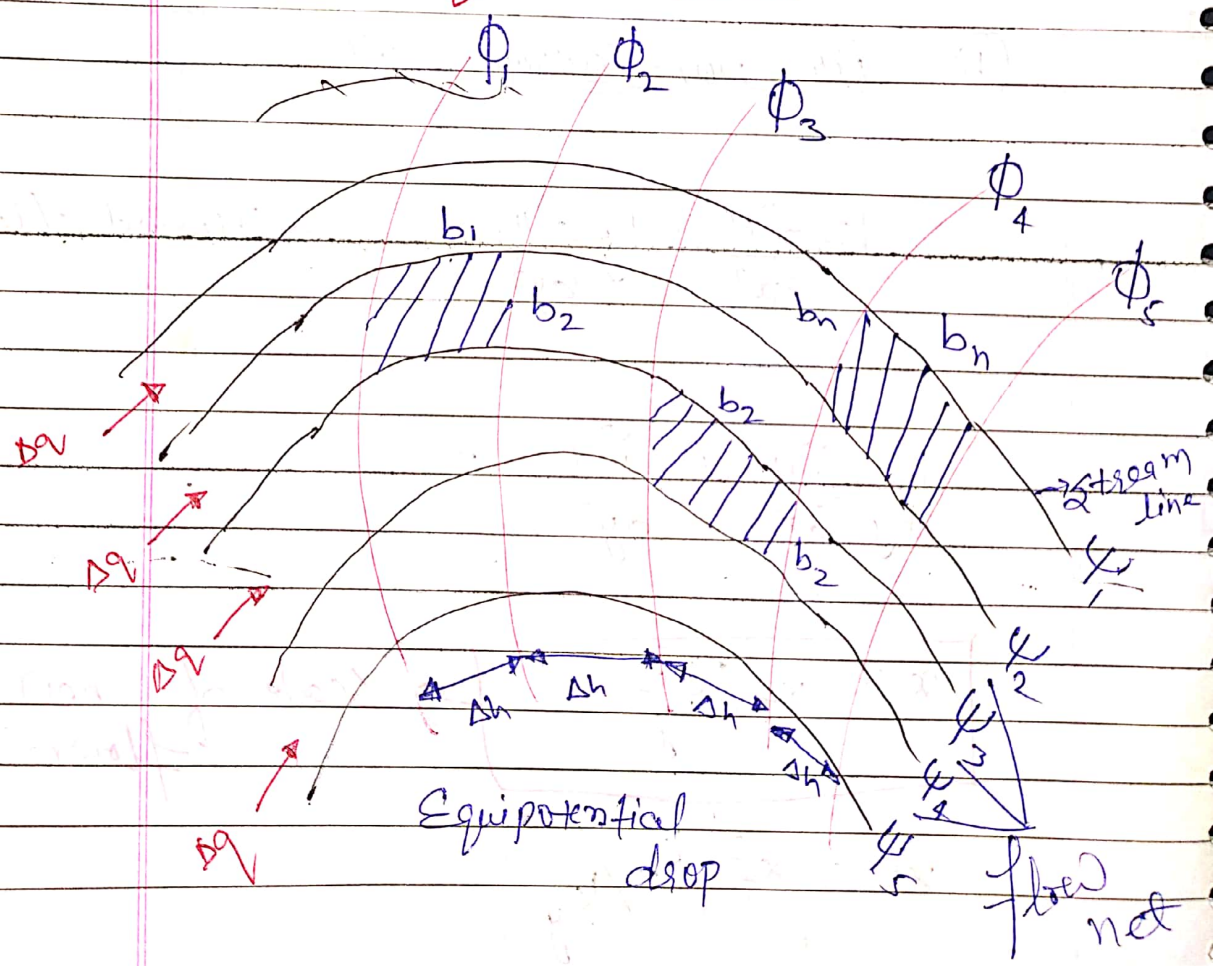
$$x^2 = x_T^2 \left(\frac{k_x}{k_y} \right)$$

$$\frac{k_x}{k_y} \frac{\partial^2 H}{\partial x_T^2} + \frac{\partial^2 H}{\partial y^2} = 0$$

$$\frac{\partial^2 H}{\partial x_T^2} + \frac{\partial^2 H}{\partial y^2} = 0$$

We can say that flow net plotted on scale x_T & y then it will be a square flow net

* Properties of Flow net :-



ϕ = equipotential function

ψ = stream line function

$$\phi_1 > \phi_2 > \phi_3 > \phi_4 > \phi_5$$

(i) ϕ line or ψ line cut each other orthogonally (90°)

(ii) Two ϕ -line or two ψ line never cross each other.

(iii) The flow field are square, for homogenous and isotropic where as non-isotropic these are rectangular may be curvilinear or linear.

(4) Head loss through each flow field is equal and is called equipotential drop

Let total head loss b/w U/s & d/s section

$$= H_L = H_1 - H_2$$

if N_d = No. of equipotential drop.

$$\Delta h = \frac{H_L}{N_d}$$

(5) The discharge through each flow channel is Δq and remains constant. that means no component of flow is perpendicular to $\psi = \text{line}$.

discharge per unit width of soil.

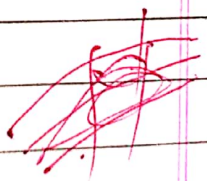
$$\Delta q = \psi_1 - \psi_2 = \psi_2 - \psi_3 = \psi_3 - \psi_4 = \psi_4 - \psi_5 = \dots$$

If $N_f = N_o$ of flow channels in a flow net & $Q = \text{total discharge}$

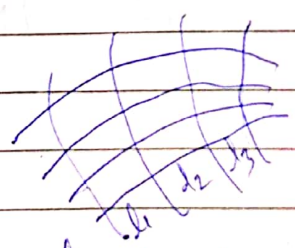
$$\Delta q = \frac{Q}{N_f}$$

$$\Delta q = v_1 (b_1 \times L) = v_2 (b_2 \times L) = v_n (b_n \times L)$$

(6) In case of conversion flow net the size of the last flow field is minimum. therefore its component to check that hydraulic gradient at exit (i.e) should be less than (i.e)



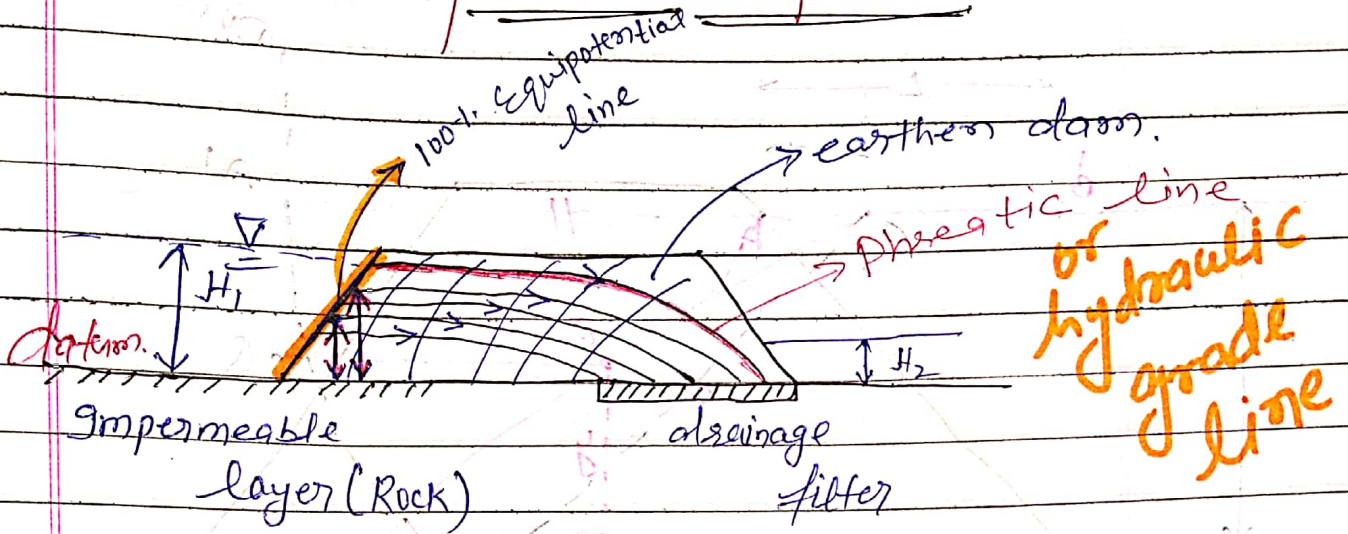
$$i_e = \frac{\Delta h}{l_e}$$



$l_e = \text{length of exit flow field}$

④ A given flow net remain same even if the direction of flow is reversed. because flow net depend only on boundary conditions not on direction of flow.

→ Discharge through earthen dam (drainage filter is provided): —



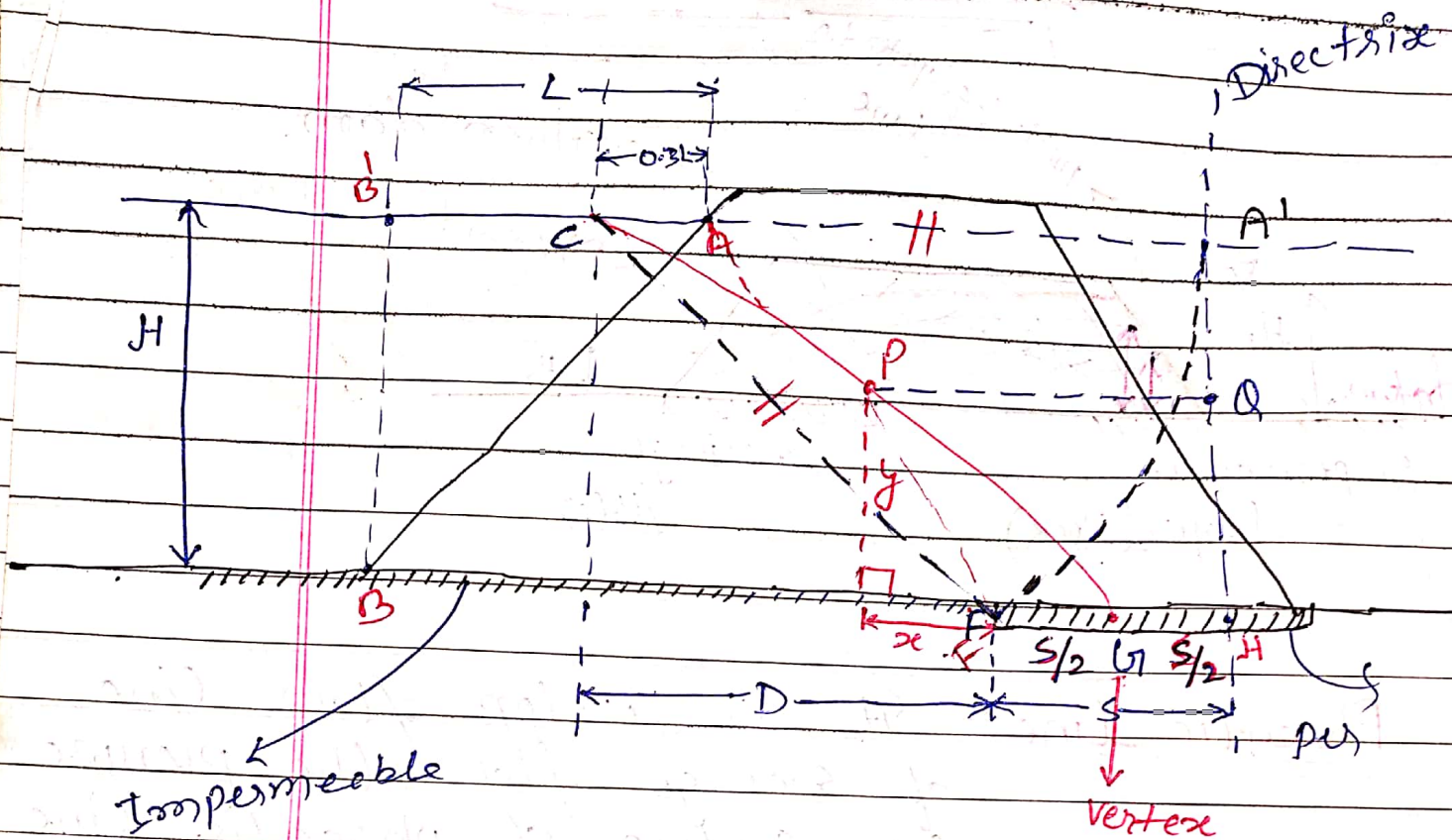
Phreatic line :- It is a top flow line of seepage flow. The pressure on surface of phreatic line is atmospheric. It is also called hydraulic grade line. Below phreatic line, seepage flow is under hydrostatic pressure.

100% equipotential line :- The upstream wetted surface of earthen dam is 100% equipotential line.

$$H_1 - y_1 + y_1 = H_1$$

$$H_1 - y_2 + y_2 = H_1$$

Procedure to draw phreatic line



$$FH = lH$$

$FH = \text{focal length} = S$

$C =$ Entry point of base parabola

$F =$ Focus of parabola

① phreatic line follows ~~path~~ a path of a parabola whose focus is the junction of permeable and impermeable surface.

② The entry point of parabola is located at point C

on water surface where AC is equal to $0.3L$

(iii) To find discrete and ARC is draw with radius CF which cuts CA produce at A'

s.e

$$CA' = CF$$

(iv) Draw vertical line through A' to meet filter at H.

A'H is the discrete

FH is focal length

mark G such that $FG = GH = \frac{S}{2}$

for any point 'p'

$$PF = \sqrt{x^2 + y^2} = PG$$

$$\sqrt{x^2 + y^2} = S + x$$

$$x^2 + y^2 = S^2 + x^2 + 2Sx$$

$$y = \sqrt{S^2 + 2Sx}$$

→ eqⁿ of parabola
(phreatic line)

If D is horizontal distance b/w energy point (e) and focus

$$CF = CA'$$

$$\sqrt{D^2 + H^2} = D + S$$

$$S = \sqrt{D^2 + H^2} - D$$

Seepage discharge through body of dam per unit width of dam is

$$q = k_s A$$

$$q = k \cdot \frac{dy}{dx} (y \times 1)$$

from eqn - (1) $y = \sqrt{s^2 + 2sx}$

$$\frac{dy}{dx} = \frac{1}{2} \frac{2s}{\sqrt{s^2 + 2sx}}$$

$$q = k \cdot \frac{dy}{dx} \cdot y$$

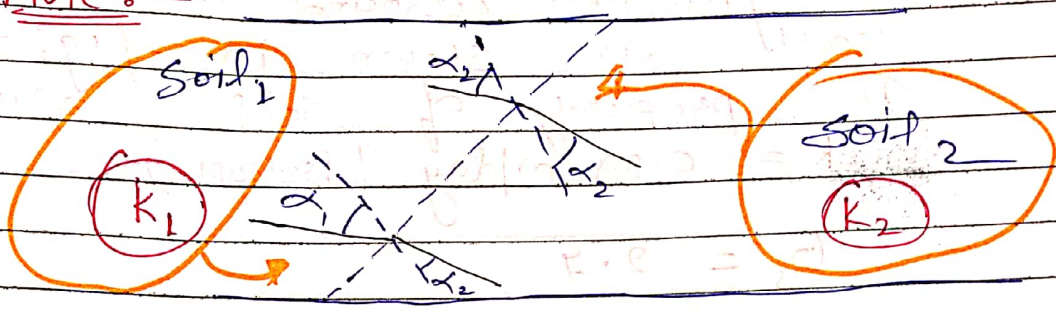
$$q = k \left(\frac{s}{\sqrt{s^2 + 2sx}} \right) \sqrt{s^2 + 2sx}$$

$$q = k_s$$

$$k = \sqrt{k_{xz} \cdot k_y} \quad \text{--- (2D)}$$

$$k = \sqrt[3]{k_{xz} \cdot k_y \cdot k_z} \quad \text{--- (3D)}$$

Note :-



$$\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{k_1}{k_2}$$

Q → A ~~Hom~~ Homogeneous and Isotropic dam is 20m high and is constructed on a foundation. The $k_x = 4.8 \times 10^{-8}$ m/s and $k_y = 1.6 \times 10^{-8}$ m/s. Water level on up stream is 18m and down stream is 0m. The shape factor of flow net is $2/9$. Estimate the seepage discharge through unit length of dam.

$k_x = 4.8 \times 10^{-8}$ m/s
 $k_y = 1.6 \times 10^{-8}$ m/s

$\frac{N_f}{N_d} = \frac{2}{9}$

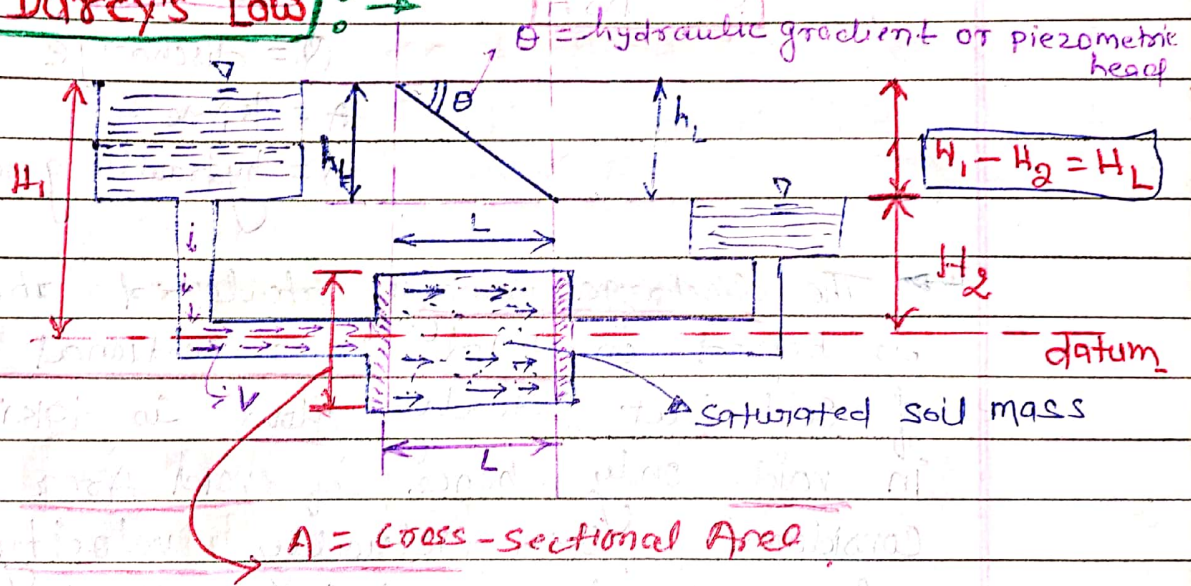
$H_1 = 18$ m
 $H_2 = 0$ m.

$q = \frac{N_f}{N_d} \times \sqrt{k_x \cdot k_y} \times H$
 $= \frac{2}{9} \times \sqrt{4.8 \times 10^{-8} \times 1.6 \times 10^{-8}} \times 18$
 $= 11.08 \times 10^{-8}$
 $\text{m}^3/\text{sec}/\text{m}$

Permeability

- ↳ Knowledge of Permeability is essential in Settlement of buildings, yields of wells, Seepage through and below the earth surface.
- ↳ It controls the hydraulic stability of soil masses.
- ↳ The permeability of soils is also required in the design of filters used to prevent piping in hydraulic structures and subgrade drainage.
- ↳ rate of consolidation of compressible soils.

Darcy's Law



Head Loss (H_L) = $H_1 - H_2$

$\tan \theta = \frac{h_L}{L} = i = \text{hydraulic gradient line or Piezometric head}$

A/c to Darcy's Law :-

↳ A/c to Darcy's law for laminar condition in saturated soil mass the discharge velocity is directly proportional hydraulic gradient

$$v_d \propto i$$

k = Co-efficient of permeability
m/sec or cm/sec.

$$v_d = k i$$

$$\frac{Q}{A} = k i$$

$$Q = k i A$$

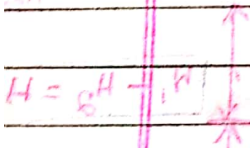
$$Q = A \cdot v$$

$$v = \frac{Q}{A}$$

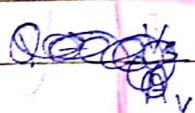
Q = discharge

A = cross-sectional area

i = hydraulic gradient = $\frac{h_L}{L}$



↳ The discharge velocity calculated above is based on total cross-sectional area of soil. but actually flow is taking place in void only. hence if void area is considered then actually velocity of flow is true velocity and is called seepage velocity (v_s)



$$v_s = \frac{Q}{A_v}$$

$$Q = A v$$

$$Q = v_s A_v$$

$$A v = v_s A_v$$

$$v_s = \frac{A \cdot v}{A_v}$$

$$v_s = \frac{A \cdot L \cdot v_d}{A_v \cdot L} = \frac{v_d}{\frac{A_v}{A}} = \frac{v_d}{n} = v_s$$

$$V_s = \frac{V_d}{n} \rightarrow \text{porosity}$$

↳ $n < 1$, V_s is always greater than V_d .
 $V_s > V_d$
 but always used V_d

Typical values of permeability are as listed in the table.

Soil	Permeability (K)
Gravel	$> 2 \text{ cm/sec}$
Sand	10^{-3} cm/sec
Silt	$10^{-3} - 10^{-7} \text{ cm/sec}$
clays	$< 10^{-7} \text{ cm/sec}$

Co-efficient of percolation (K_p)

$$V_s = K_p i$$

$$\frac{V_d}{n} = K_p i$$

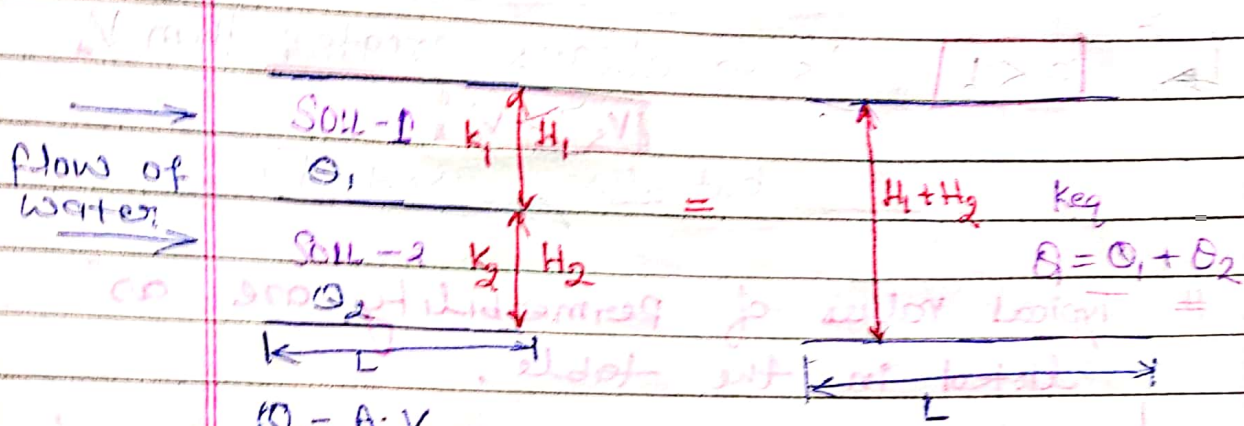
$$\frac{K_p}{n} = K_p$$

$$K_p = \frac{K}{n}$$

Set for the piezometer
 like parameter to

Permeability (k) of stratified soil.

(A) When flow is parallel to the bed.



$$Q = A \cdot V$$

$$= A k S$$

$$= A k \frac{h_L}{L}$$

$$Q = Q_1 + Q_2$$

$$Q = k_{eq} \frac{h_L}{L} (H_1 + H_2)$$

Now

$$Q = Q_1 + Q_2$$

$$Q_1 = k_1 \frac{h_{L1}}{L} (H_1)$$

+

$$Q_2 = k_2 \frac{h_{L2}}{L} (H_2)$$

$$k_{eq} \frac{h_L}{L} (H_1 + H_2) = k_1 \frac{h_{L1}}{L} (H_1)$$

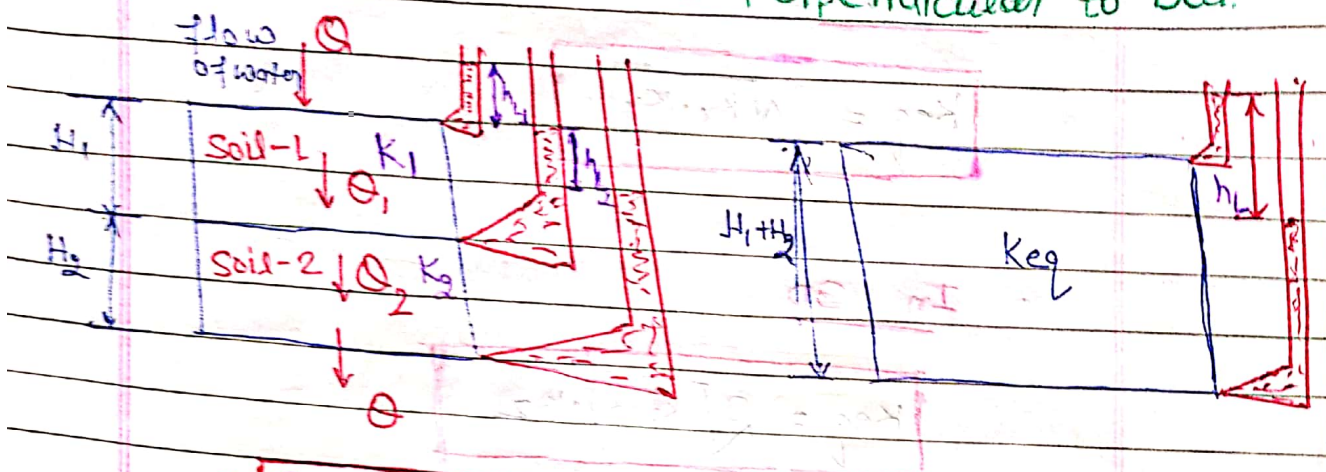
$$+ k_2 \frac{h_{L2}}{L} (H_2)$$

$H_L = H_{L1} = H_{L2}$
Set the piezometer height at same of soil

$$k_{eq} = \frac{k_1 H_1 + k_2 H_2}{H_1 + H_2}$$

$$Q = Q_1 + Q_2$$

B When water flow is perpendicular to bed.



* $Q_1 = Q_2 = Q$

$h_L = h_{L1} + h_{L2}$

$Q_1 = K_1 i_1 A = K_1 \frac{h_{L1}}{H_1} \cdot A \Rightarrow h_{L1} = \frac{Q_1 H_1}{K_1 A}$

$Q_2 = K_2 i_2 A = K_2 \frac{h_{L2}}{H_2} \cdot A \Rightarrow h_{L2} = \frac{Q_2 H_2}{K_2 A}$

$Q = K_{eq} i \cdot A = K_{eq} \frac{h_L}{(H_1 + H_2)} \cdot A \Rightarrow h_L = \frac{Q (H_1 + H_2)}{K_{eq} A}$

$h_L = h_{L1} + h_{L2}$

$\frac{Q_1 H_1}{K_1 A} + \frac{Q_2 H_2}{K_2 A} = \frac{Q (H_1 + H_2)}{K_{eq} A}$ ∵ $Q_1 = Q_2 = Q$

$K_{eq} = \frac{H_1 + H_2}{\frac{H_1}{K_1} + \frac{H_2}{K_2}}$

In 2D

$$K_{eq} = \sqrt{K_x \cdot K_y}$$

In 3D

$$K_{eq} = \sqrt[3]{K_x \cdot K_y \cdot K_z}$$

Factor Affecting Permeability :->

① Particle Size :-> If void ratio is same the coarse soil are more permeable.

A/c to Allen Hazen :-

Co-efficient of permeability of soil is proportional to the square of a representative particle size

$$k \propto D_{10}^2$$

$$K = c D_{10}^2$$

K = Permeability of soil, cm/sec

D₁₀ = effective size of particle, mm

↳ c = constant (0.40 - 1.02) if D₁₀ in mm

↳ c = 100 if D₁₀ in cm

↳ Effect of void ratio

② EFFECT OF VOID RATIO :-> If particle size is same then loose soil are more permeable than dense soil.

A/c to Taylor :-

$$k \propto \frac{e^3}{1+e} \quad \text{or} \quad k \propto e^2$$

$$k = \frac{c e^3}{1+e}$$

$$K = c e^2$$

e = shape factor.

shape factor

$$\frac{k_1}{k_2} = \frac{c_1 \frac{e_1^3}{1+e_1}}{c_2 \frac{e_2^3}{1+e_2}}$$

c_1 & c_2 are depends upon placing of soil grain and shape characteristics of pores

For sand :->

c changes only slightly with void ratio i.e. $c_1 = c_2$

$$\frac{k_1}{k_2} = \frac{\frac{e_1^3}{1+e_1}}{\frac{e_2^3}{1+e_2}}$$

$$\frac{k_1}{k_2} = \frac{e_1^3}{e_2^3}$$

FOR silt and clay :->

$$\frac{\log_{10} k_1}{\log_{10} k_2} = \frac{e_1}{e_2}$$

③ Shape Particle :-> Effect of shape of particle is expressed in terms of Specific surface area (SA)

Permeability of soil is inversely proportional to square of SA

$$K \propto \frac{1}{SA^2}$$

$$SA = \frac{\text{Surface Area}}{\text{Volume}}$$

eg:-

Sphere :-

FOR SAME VOID RATIO
Rounded $\rightarrow K \uparrow, SA \downarrow$
Angular $\rightarrow K \downarrow, SA \uparrow$

$$SA = \frac{4\pi r^2}{\frac{4}{3}\pi r^3} = \frac{3}{r}$$

④ Degree of saturation : → Partially saturated soil the air void present create resistance in the flow of water. Hence less permeability.

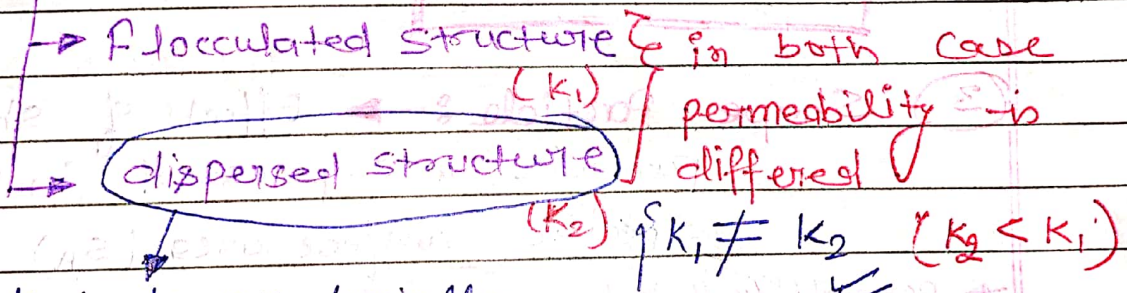
$K \propto S$

$S_{Full} \rightarrow K \uparrow$
 $S_p \rightarrow K \downarrow$

⑤ Structure of soil : →

↳ In stratified soil, permeability is more in parallel to the bed and permeability is less in perpendicular to the bed.

Two clays of same void ratio



K is decreases basically bec of reduction in size of void available for flow.

⑥ Impurities : → if impurities is more then permeability is decreases.

↳ More is impurities less

⑦ Effect of Permeant (its viscosity and temperature)

↳ Permeability of soil is also depends upon the density (or unit weight) or its viscosity

γ or γ μ

so

$\mu \propto \frac{1}{\text{Temp}}$

$k \propto \frac{\gamma}{\mu}$

$\gamma =$ Unit wt. of fluid (generally water)

$\mu =$ viscosity of fluid.

$$\frac{k_1}{k_2} = \frac{\frac{\gamma_{u1}}{\mu_1}}{\frac{\gamma_{u2}}{\mu_2}} = \frac{\gamma_{u1} \cdot \mu_2}{\gamma_{u2} \cdot \mu_1}$$

↳ k is more affected by change in viscosity than change in unit weight.
 because unit weight of water does not change much over large range of temp.

$$\frac{k_1}{k_2} = \frac{\mu_2}{\mu_1}$$

↳ Greater is the viscosity lower is the permeability

• A/c to IS: 2720 Part 17; Permeability of soil shall be expressed at 27°C

$$k_{27} = \frac{\mu_T}{\mu_{27}} k_T$$

$k_{27} =$ value of k at 27°C

$k_T =$ " " " " T°C

$\mu_{27} =$ viscosity of permeant at 27°C

$\mu_T =$ " " " " T°C

⑧ Effect of effective stress: →

↳ As effective stress increases, void ratio decrease and consequently permeability decrease.

$$\bar{\sigma} \uparrow \rightarrow e \downarrow \rightarrow k \downarrow$$

⑨ Adsorbed water: → more is absorption capacity less is the permeability

NOTE →

Sand $\leftarrow k > 10^{-3}$ → determined by constant head method.

$10^{-3} < k < 10^{-2}$ → determined by falling head.

$k < 10^{-3}$ → determined by consolidation formula

Coefficient of absolute permeability: k_o or intrinsic permeability. $\left\{ \begin{array}{l} \text{specific permeability} \\ \text{intrinsic permeability} \end{array} \right.$

↳ This parameter is independent of property of fluid and depends only on soil property

$$k_o = \frac{k \cdot \mu}{\gamma_w}$$

k_o In m^3 or cm^2 or Darcy

$$1 \text{ Darcy} = 0.987 \times 10^{-8} \text{ cm}^2$$

Determination of Co-efficient of Permeability

LABORATORY Method

- Constant-head Permeability test :-
- variable/falling head Permeability test
- capillary - permeability test.

FIELD METHOD

- Pumping-out test (large area required)
- Pumping-in test (small area required)

INDIRECT METHOD

- Computation from the particle size and its specific surface (used when particle size known)
- Computation from the consolidation test data. (used when co-efficient of volume change (m_v) has been determined by consolidation test on the soil.)

LABORATORY METHODS

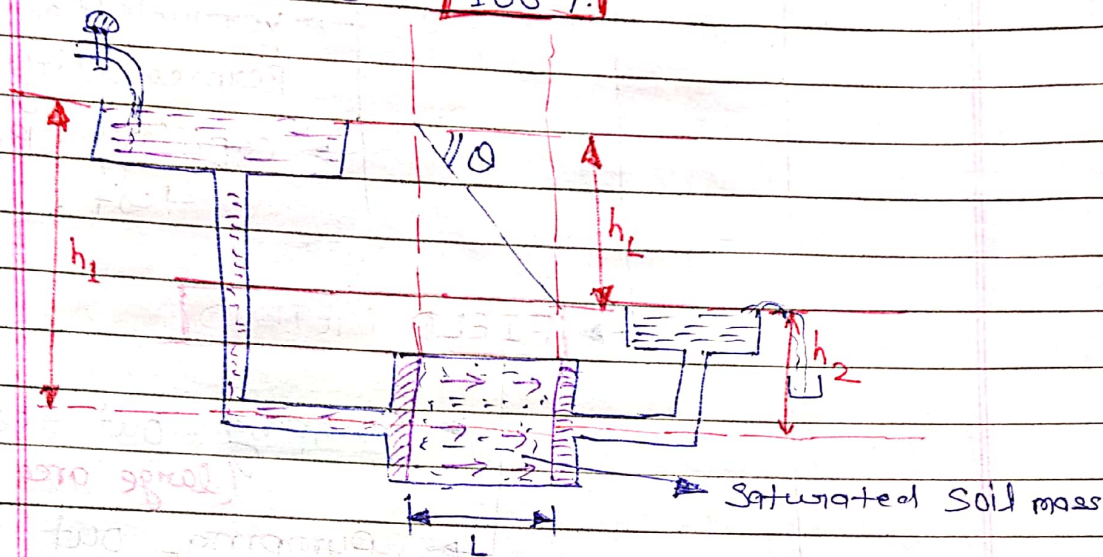
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* CONSTANT HEAD PERMEABILITY TEST :-

↳ This test is generally used for coarse soil i.e. Sand.

↳ Degree of saturation of soil should be 100%.



↳ Let

Quantity of collected in graduated vessel in Time t is Q

Hence

$$\text{Discharge (q)} = \frac{\text{Quantity (Q)}}{\text{Time (t)}}$$

q = discharge

A = c/s Area

L = length of flow

h_L = difference in monometer

$$\text{Discharge (q)} = \frac{\text{Quantity (Q)}}{\text{Time (t)}}$$

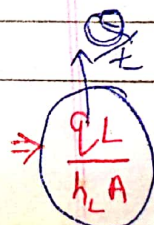
$$q = \frac{Q}{t}$$

q also know

$$q = k \cdot s \cdot A$$

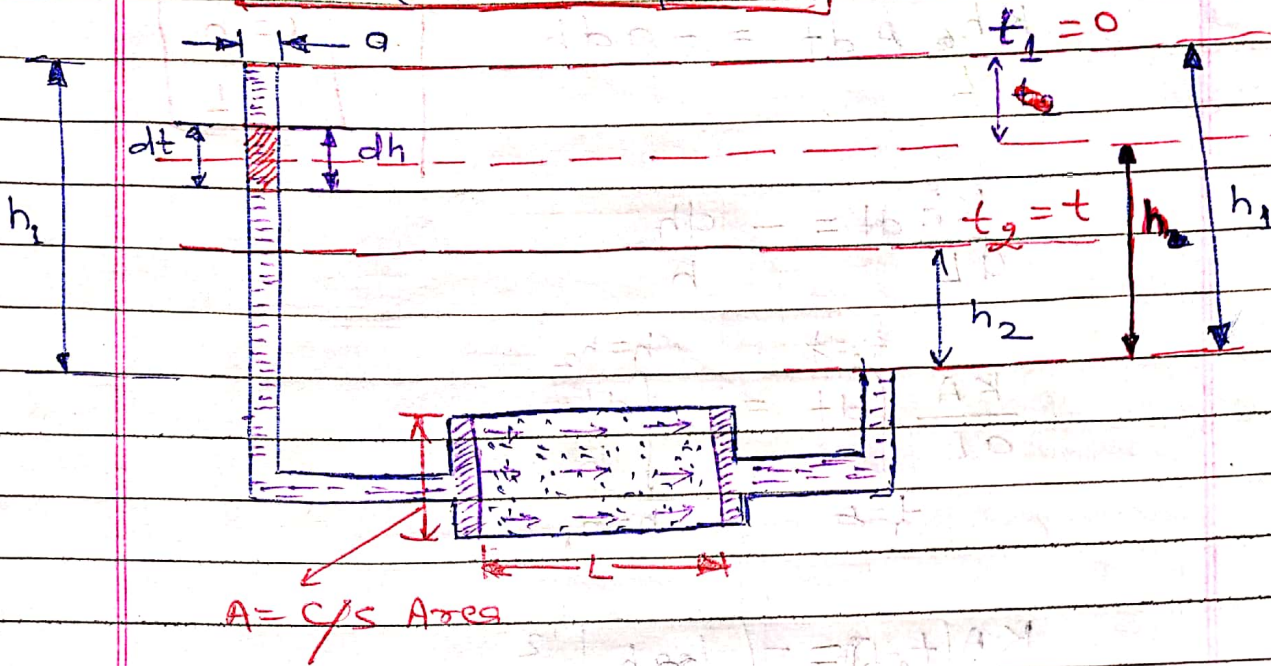
$$\frac{Q}{t} = k \frac{h_L A}{L} \Rightarrow k = \frac{QL}{h_L A t}$$

$$k = \frac{QL}{h_L A t}$$



FALLING HEAD PERMEABILITY TEST →

↳ This test is generally used for **fine soil**
undisturbed specimen



Let

time $t_1 = 0$ → Head difference b/w up & down stream level be h_1

time $t_2 = t$ → Head difference b/w up & down stream level be h_2

Let,

at any intermediate stage the head difference h and it fall by dh in dt time.

Now

volume of water fall in time interval dt

$Q = \text{discharge}$

~~$V = Q \times dt$~~

$V = Q \times dt$

$Q = \frac{\text{Volume}}{\text{time}}$

Volume of water pass through soil. at dh .

$V = -a \times dh$

By continuity

$Q \times dt = -a \times dh$
 $kSA \times dt = -a \times dh$

$v = ks$
 $\frac{Q}{A} = k \cdot i$
 $Q = k \cdot i \cdot A$

$$k \rho A dt = -a dh$$

$$0 = \frac{k h_0}{L} A dt = -a dh$$

$$\frac{a}{L} = h$$

$$t = \frac{k A}{a L} dt = - \frac{dh}{h}$$

$$\frac{k A}{a L} \int_{t_1}^{t_2} dt = - \int_{h_1}^{h_2} \frac{dh}{h}$$

$$\frac{k A}{a L} (t_2 - t_1) = - \left[\log h \right]_{h_1}^{h_2}$$

$$\frac{k A}{a L} (t_2 - t_1) = - \left[\log h_2 - \log h_1 \right]$$

$$\frac{k A}{a L} (t_2 - t_1) = \ln h_1 - \ln h_2$$

$$\frac{k A}{a L} (t_2 - t_1) = \ln \frac{h_1}{h_2}$$

$$k = \frac{a L}{A (t_2 - t_1)} \ln \frac{h_1}{h_2}$$

$$k = \frac{2.303 a L}{A (t_2 - t_1)} \log_{10} \frac{h_1}{h_2}$$

NOTE → In above test conducted in two stages. i.e. in first stage head fall from h_1 to h_2 in t time and its second stage head falls from h_2 to h_3 in same time t then.

$$K = \frac{2.303 aL}{At} \log_{10} \frac{h_1}{h_2} = \frac{2.303 aL}{At} \log_{10} \frac{h_2}{h_3}$$

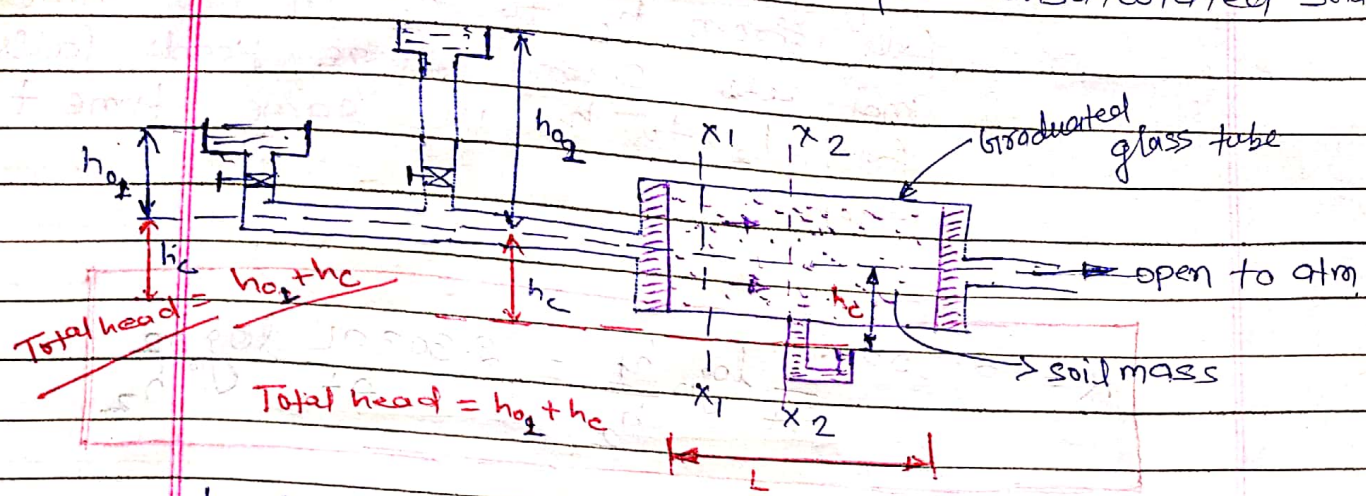
$$\log_{10} \frac{h_1}{h_2} = \log_{10} \frac{h_2}{h_3}$$

$$\frac{h_1}{h_2} = \frac{h_2}{h_3}$$
$$h_2^2 = h_1 h_3$$

$$h_2 = \sqrt{h_1 \cdot h_3}$$

Capillary Permeability test :-

↳ This test is used for unsaturated soil.



↳ In first stage smaller head h_{01} is connected to soil mass and water moves from x_1 to x_2 in time t_1 to t_2 .

$$\frac{x_2^2 - x_1^2}{t_2 - t_1} = \frac{2K}{S \cdot n} (h_{01} + h_c)$$

↳ In 2nd stage larger head h_{02} is connected to soil mass let water surface moves from x'_1 to x'_2 in the time t'_1 to t'_2 .

$$\frac{x'^2_2 - x'^2_1}{t'_2 - t'_1} = \frac{2K}{S \cdot n} (h_{02} + h_c)$$

- K = Co-efficient of permeability
- S = Degree of saturation
- n = porosity
- h_c = capillary head.

FIELD TEST METHODS

Pumping out test :->

(a) Unconfined aquifer :->

(1) Well is penetrated up to the bottom of aquifer such that water comes only from side of the well not from the bottom.

(2) Study state seepage is stabilized towards the wells.

this results in the water table being drawn down to form a cone of depression.

(3) The hydraulic gradient at distance from the centre of well is assumed to be curved.

* Drawdown :-> Drawdown at some distance r from the centre of well is equal to fall in water level from original ground water level.

Radius of influence :-> Distance from the centre of well such that drawdown is equal to zero

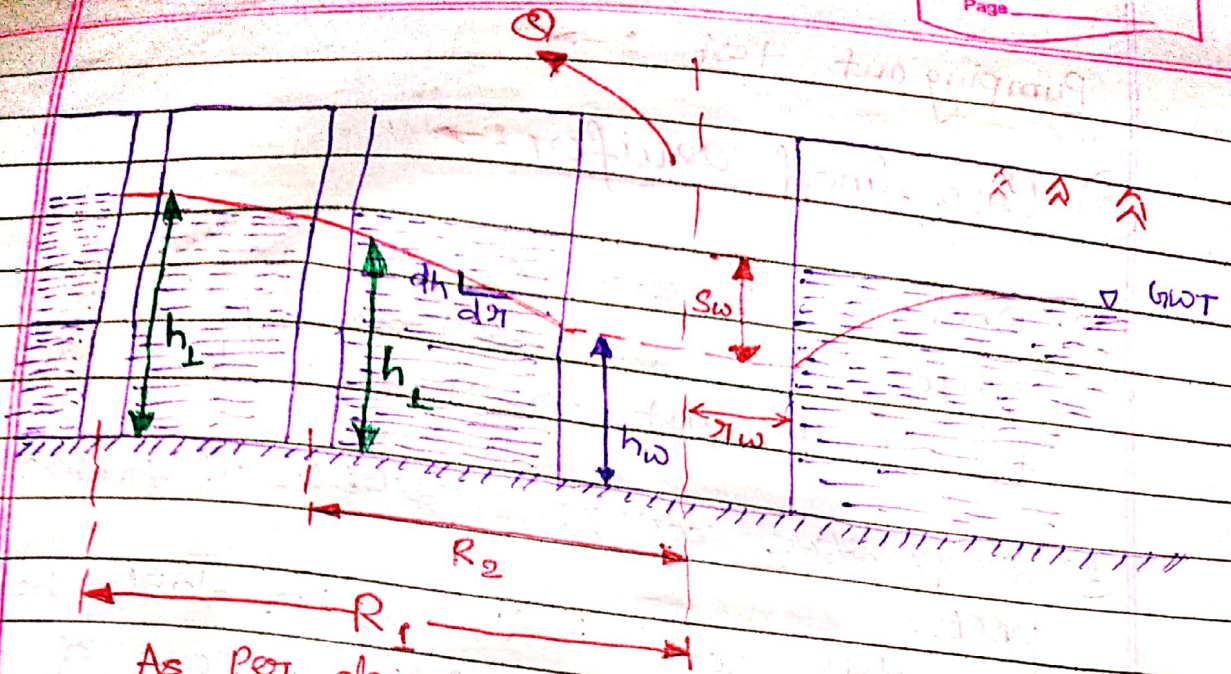
-> Radius of influence varies b/w 150 to 300m
A/c to Sichardt

$$R = 3000d \sqrt{K}$$

d = drawdown of well (m)

R = Radius of Influence (m)

K = m/sec .



As per Darcy

$$Q = k s A$$

$$Q = k \cdot \frac{dh}{dr} \cdot A \quad \rightarrow \quad A = 2\pi r h$$

$$k = \frac{Q \cdot dr}{2\pi r h dh}$$

$$k \int_{h_1}^{h_2} h dh = \int_{R_1}^{R_2} \frac{Q}{2\pi} \cdot \frac{dr}{r}$$

$$k \left(\frac{h_2^2 - h_1^2}{2} \right) = \frac{Q}{2\pi} \ln \left(\frac{R_2}{R_1} \right)$$

$$k = \frac{Q}{\pi (h_2^2 - h_1^2)} \ln \left(\frac{R_2}{R_1} \right)$$

$$R_2 \rightarrow r_w$$

$$h_2 \rightarrow h_w$$

$$k = \frac{Q}{\pi (h_w^2 - h_1^2)} \ln \left(\frac{R_w}{R} \right)$$

$$k = \frac{Q \times 2.303}{\pi (h_w^2 - h_1^2)} \log_{10} \frac{R_w}{R}$$

(b) Confined Aquifer: \rightarrow

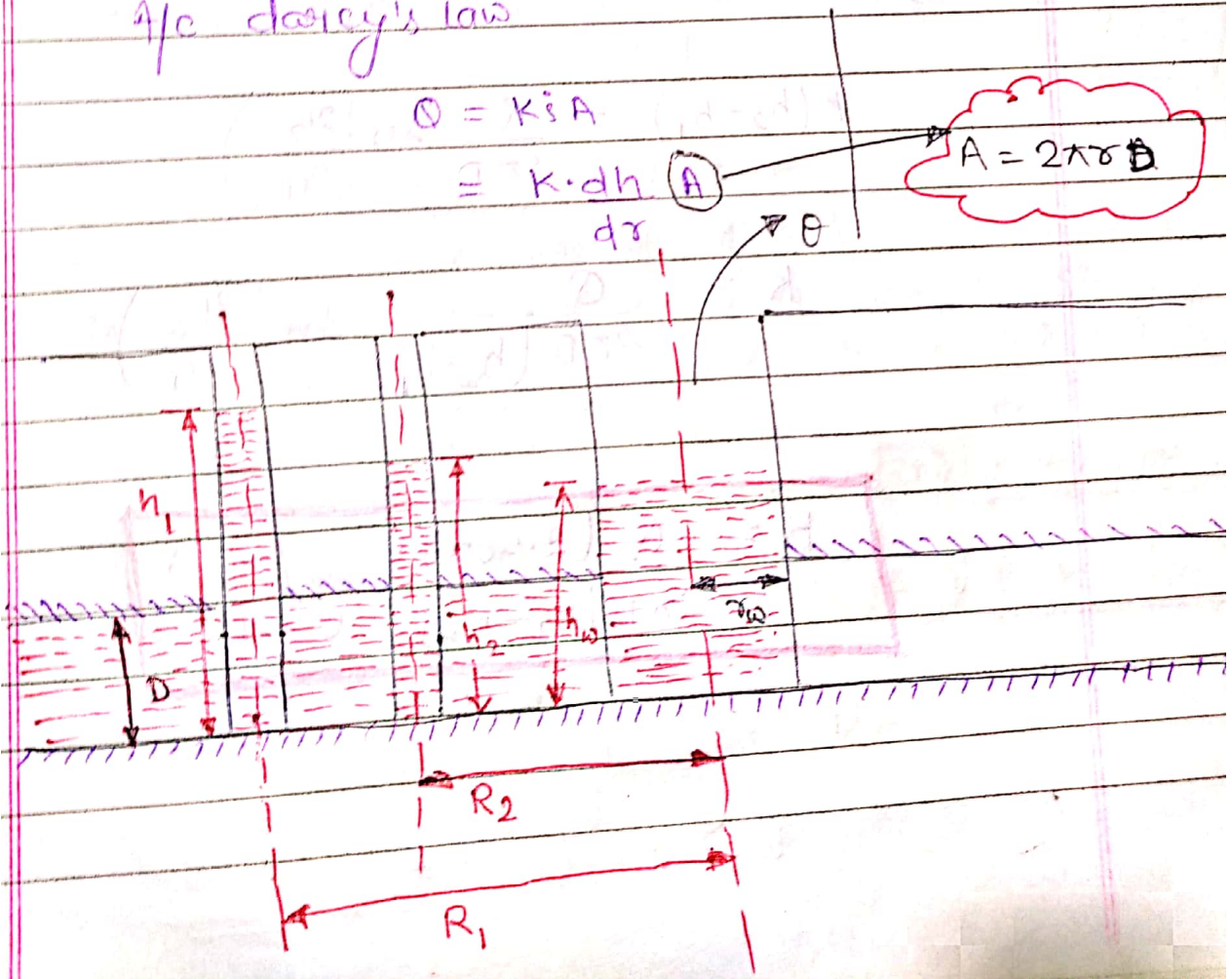
A confined flow condition occurs when the aquifer is confined both above and below by impermeable strata.

A/c Darcy's law

$$Q = k_i A$$

$$= k \cdot dh \cdot A$$

$$A = 2\pi r dr$$



$$Q = k \frac{dh}{dr} \cdot A$$

$$Q = k \frac{dh}{dr} (2\pi r D)$$

$$k = \frac{Q \cdot dr}{r \cdot 2\pi D \cdot dh}$$

$$k dh = \frac{Q}{2\pi D} \cdot \frac{dr}{r}$$

$$k \int_{h_1}^{h_2} dh = \frac{Q}{2\pi D} \int_{r_1}^{r_2} \frac{dr}{r}$$

$$k(h_2 - h_1) = \frac{Q}{2\pi D} \ln \left(\frac{r_2}{r_1} \right)$$

$$k = \frac{Q}{2\pi D (h_2 - h_1)} \ln \left(\frac{r_2}{r_1} \right)$$

$$k = \frac{Q (2.303)}{2\pi D (h_2 - h_1)} \log \left(\frac{r_2}{r_1} \right)$$

INDIRECT METHOD

① **Kozeny - Carman Equation** :- \rightarrow

$$K = \frac{1}{K_K} \cdot \frac{\gamma_w \cdot e^3}{\mu (1+e)} \cdot \frac{1}{S_A^2}$$

\rightarrow unit wt of water.

\rightarrow Shape factor for Kozeny - Carman

μ = Co-efficient of viscosity

S_A = shape factor Specific surface Area
(ie sp. area per unit vol. of solid)

②

Specific Area for spherical particle

$$S_A = \frac{\pi D^2}{\frac{\pi D^3}{6}} = \frac{6}{D}$$

\rightarrow If particle size b/w a & b the sp. surface Area = $\frac{6}{\sqrt{ab}}$

② **Allen Hazen's Formula** :- \rightarrow

\rightarrow Co-efficient of permeability of soil is proportional to the square of representative particle size.

$$K = C D_{10}^2$$

\rightarrow $C = 100$ to 150 if D_{10} is in **cm** ($K \rightarrow$ cm/sec)

\rightarrow $C = 1$ to 1.5 if D_{10} is in **mm** ($K \rightarrow$ cm/sec)

③ :-> London's Formula :->

$$\log_{10} (K S_A^n) = a + bn$$

n = porosity

a & b = constant

S_A = sp. Surface Area.

④ Consolidation Formula :->

↳ This method is suitable for fine grained soil. whose $K < 10^{-7}$ cm/sec for which permeability test can not be conducted in laboratory

$$K = C_v \gamma_w m_v$$

C_v = Co-efficient of consolidation

m_v = Co-efficient of volume change.

Q → Estimate the value of co-efficient of permeability for a uniform graded sand of size $D_{10} = 0.15$ mm obtained from sieve analysis $C_u = 2.67$

by Allen Hazen formula

$$K = C D_{10}^2$$

Let

$$C = 1 \text{ bez } D_{10} \text{ in mm}$$

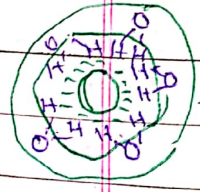
$$K = 1 \times (0.15)^2 = 0.0225 \text{ cm/sec.} \\ = 0.225 \text{ mm/sec}$$

SHEAR STRENGTH OF SOIL

- Soil will always fail in shearing, what be the type of loading.
- It never occurs by crushing of soil particles.
- Shear strain of soil is the resistance offered by the soil grains against shear deformation.
- Failure occurs when shear stress at any point is exceeded by ~~strength~~ shear strength of that point.
- Soil may derive its shear strength from following parameter.

(a) Frictional strength: → because of friction interlocking of soil grains

(b) Cohesion:-



(i) true cohesion: → because of electrostatic force of attraction.

(ii) Apparent cohesion → Due to negative pressure in the pores which cause attraction b/w the particles.

Methods to find out Shear Strength of SOIL

① Direct Shear test (Box Shear test)! -

This is the oldest test

② Triaxial compression test

(a) Consolidated drain test (CD)

(b) Consolidated undrain test (CU)

(c) Unconsolidated undrain test (UU)

(d) unconfined compression test

③ Torsion Balance shear test

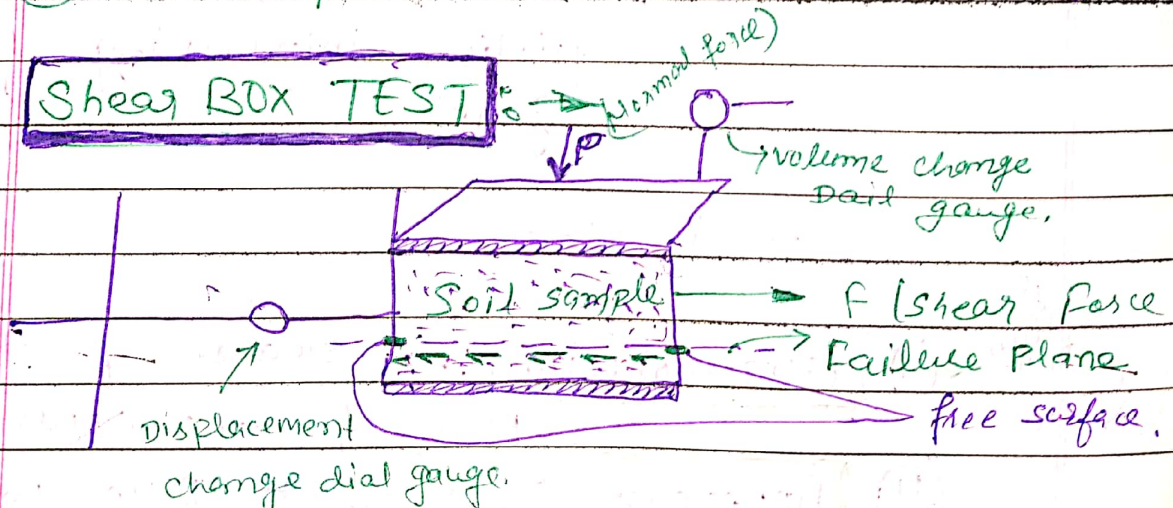
④ Ring shear test

⑤ Vane shear test

⑥ Standard penetration test (SPT)

⑦ Cone penetration test (C.P.T)

① Shear BOX TEST



Advantage! - ① Quick, inexpensive and easy to do

② Sample preparation is easy

BOX is square used

Disadvantage: — (i) Drainage cannot controlled and pore water pressure can not be measured.

(ii) This is suitable for sand and gravels. bcz they drain rapidly

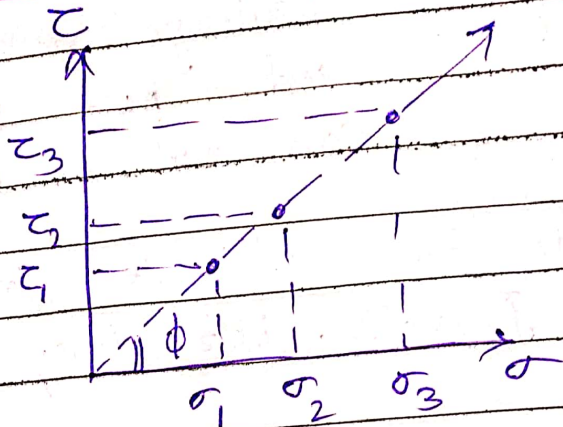
(iii) specimen fail along predetermine plane not the weakest plane

(iv) Not possible to perform undrain test which is import for clay

(v) Area of specimen under the normal load and shear does not remain constant but we take it as constant for calculation of stress.

Result of test: — various sample are tested up to failure at different normal loading and shear force required for failure is found out corresponding to normal load at time of failure.

Sample	Normal load	Shear load	Normal stress	Shear stress
①	P_1	F_1	$\frac{P_1}{A} = \sigma_1$	$\frac{F_1}{A} = \tau_1$
②	P_2	F_2	$\frac{P_2}{A} = \sigma_2$	$\frac{F_2}{A} = \tau_2$
③	P_3	F_3	$\frac{P_3}{A} = \sigma_3$	$\frac{F_3}{A} = \tau$



Straight line Equation

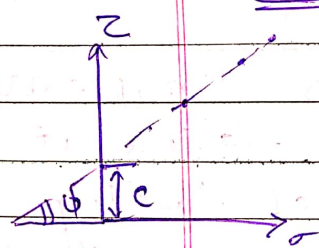
$$y = mx$$

Similarly

$$\tau = \tan \phi \cdot \sigma$$

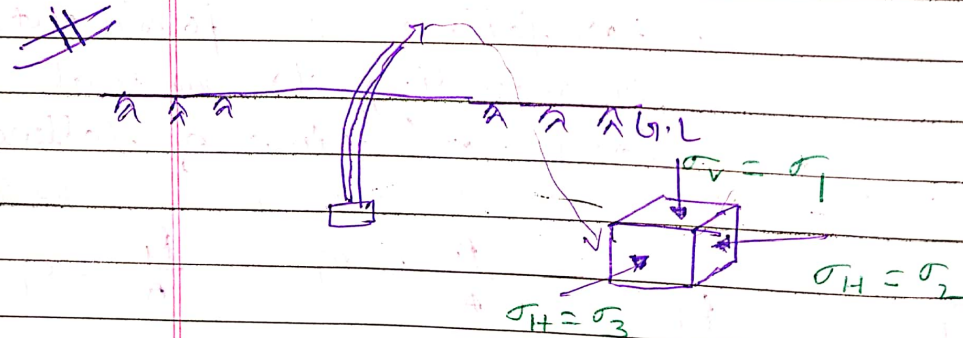
~~$\tau = \sigma \tan \phi$~~ for cohesionless soil

For Cohesive

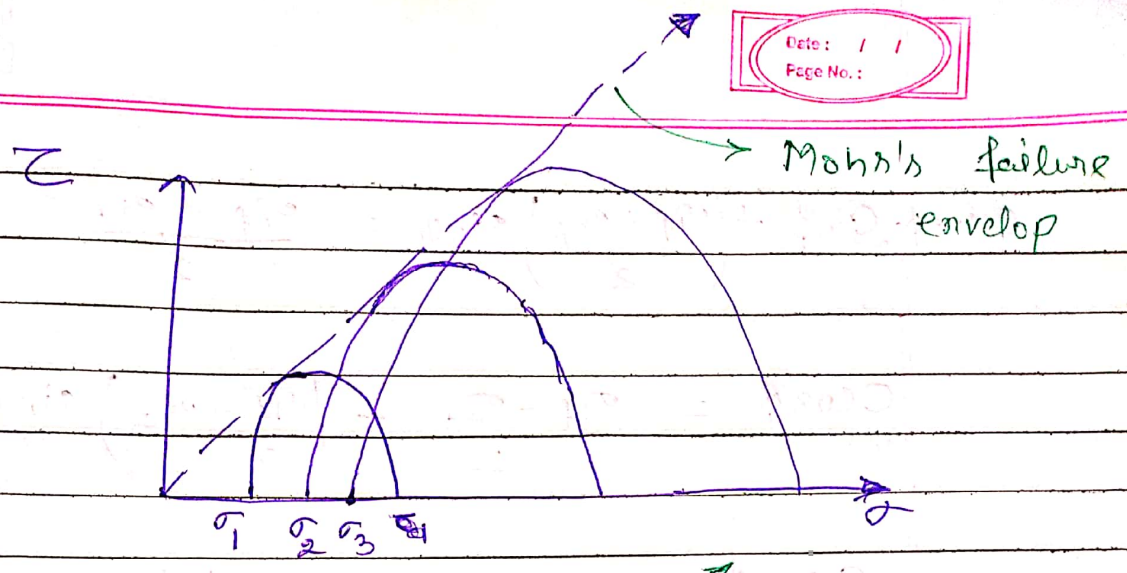


~~$\tau = \sigma \tan \phi + c$~~ (i.e. ~~$\tau = \sigma \tan \phi + c$~~)

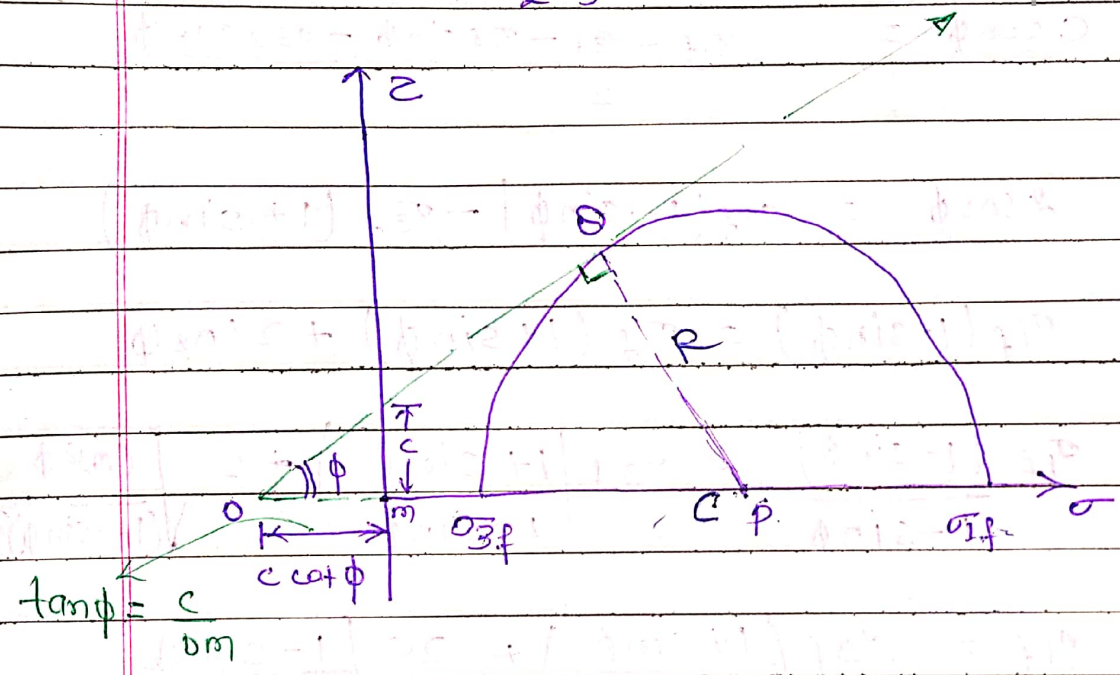
i.e. $y = mx + c$



$\sigma_1 = \text{max}^m$ principle stress
 $\sigma_2 = \sigma_3 = \text{min}^m$ principless



Mohr's failure envelope



$$\tan \phi = \frac{c}{cm}$$

$$cm = \frac{c}{\tan \phi} = c \cot \phi$$

$$OP = R = \frac{\sigma_{1f} - \sigma_{3f}}{2}$$

$$c = \frac{\sigma_{1f} + \sigma_{3f}}{2}$$

$$OP = cm + mp$$

$$= c \cot \phi + \frac{\sigma_{1f} + \sigma_{3f}}{2}$$

$$\sin \phi = \frac{OP}{OP}$$

$$\sin \phi = \frac{\frac{\sigma_{1f} - \sigma_{3f}}{2}}{c \cot \phi + \frac{\sigma_{1f} + \sigma_{3f}}{2}}$$

$$c \cot \phi \cdot \sin \phi + \left(\frac{\sigma_{1f} + \sigma_{3f}}{2} \right) \sin \phi = \frac{\sigma_{1f} - \sigma_{3f}}{2}$$

$$c \cos \phi = \frac{\sigma_{1f} - \sigma_{3f}}{2} - \left(\frac{\sigma_{1f} + \sigma_{3f}}{2} \right) \sin \phi$$

$$c \cos \phi = \frac{\sigma_{1f} - \sigma_{3f} - \sigma_{1f} \sin \phi - \sigma_{3f} \sin \phi}{2}$$

$$2c \cos \phi = \sigma_{1f} (1 - \sin \phi) - \sigma_{3f} (1 + \sin \phi)$$

$$\sigma_{1f} (1 - \sin \phi) = \sigma_{3f} (1 + \sin \phi) + 2c \cos \phi$$

$$\sigma_{1f} \frac{(1 - \sin \phi)}{1 - \sin \phi} = \sigma_{3f} \left(\frac{1 + \sin \phi}{1 - \sin \phi} \right) + 2 \frac{\cos \phi}{\sqrt{(1 - \sin \phi)^2}}$$

$$\sigma_{1f} = \sigma_{3f} \left(\frac{1 + \sin \phi}{1 - \sin \phi} \right) + 2c \frac{\sqrt{1 - \sin^2 \phi}}{\sqrt{(1 - \sin \phi)^2}}$$

$$\sigma_{1f} = \sigma_{3f} \left(\frac{1 + \sin \phi}{1 - \sin \phi} \right) + 2c \frac{1 + \sin \phi}{\sqrt{1 - \sin \phi}}$$

$$\sin \phi = 2 \sin \frac{\phi}{2} \cdot \cos \frac{\phi}{2}$$

$$\sin^2 \phi + \cos^2 \phi = 1$$

$$1 + \sin \phi = \sin^2 \phi + \cos^2 \phi + 2 \sin \frac{\phi}{2} \cdot \cos \frac{\phi}{2} = \left(\sin \frac{\phi}{2} + \cos \frac{\phi}{2} \right)^2$$

$$1 - \sin \phi = \left(\sin \frac{\phi}{2} - \cos \frac{\phi}{2} \right)^2$$

$$\frac{1 + \sin \phi}{1 - \sin \phi} = \frac{\left(\cos \frac{\phi}{2} + \sin \frac{\phi}{2} \right)^2}{\left(\cos \frac{\phi}{2} - \sin \frac{\phi}{2} \right)^2}$$

$$= \frac{\left(1 + \tan \frac{\phi}{2} \right)^2}{\left(1 - \tan \frac{\phi}{2} \right)^2}$$

$$= \tan 45^\circ + \tan \frac{\phi}{2}$$

$$\tan^2(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$$

$$1 - \tan \frac{\phi}{2} \cdot \tan 45^\circ$$

$$= \left(\tan 45^\circ + \frac{\phi}{2} \right)^2$$

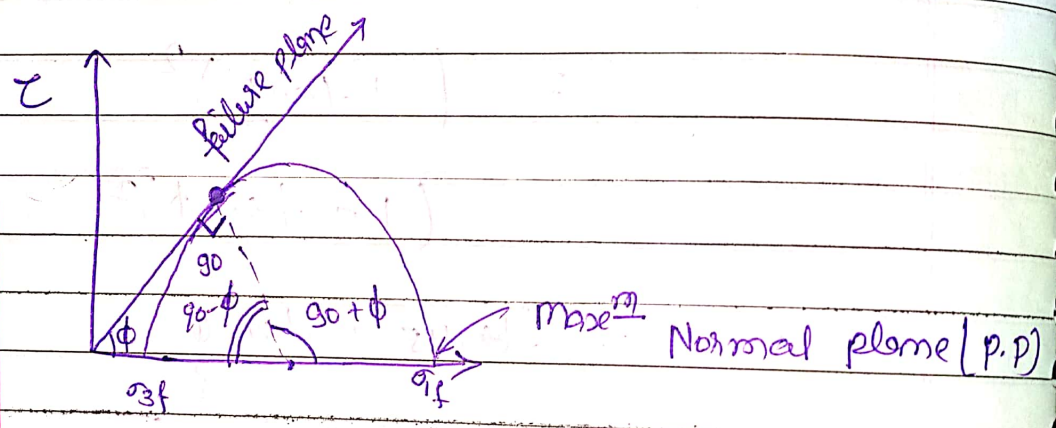
$$\sigma_{1f} = \sigma_{3f} \tan^2 \left(45^\circ + \frac{\phi}{2} \right) + 2c \tan \left(45^\circ + \frac{\phi}{2} \right)$$

* c and ϕ are collectively called shear strength parameters in which

c = Cohesive parameter
 ϕ = frictional parameter

Shear strength of soil

$$\tau = c + \sigma \tan \phi$$



→ The failure plane will be inclined at an angle of $(45 + \frac{\phi}{2})$ with the major principle plane

$$\frac{90 + \phi}{2} = 45 + \frac{\phi}{2}$$

in Mohr's circle in actual condn

→ The failure plane will be inclined at an angle of $45 - \frac{\phi}{2}$ with the minor principle plane

$$\text{in Mohr's circle } 90 - \phi = 45 - \frac{\phi}{2} \rightarrow \text{in actual}$$

Q → A direct shear test on sand gives the following results at time of failure

- ① Normal load = 288 N (P_1)
 Shear load = 173 N (F)
 C/s Area = 36 cm² (A)

Calculate - ① the angle of internal friction
 ② find the magnitude and direction of principle stress from failure plane

$$\sigma = \frac{P_1}{A} = \frac{288}{36} = 8 \text{ N/cm}^2$$

$$\tau = \frac{F}{A} = \frac{173}{36} = 4.805 \text{ N/cm}^2$$

$$\tau = \sigma \tan \phi$$

$$\frac{\tau}{\sigma} = \tan \phi$$

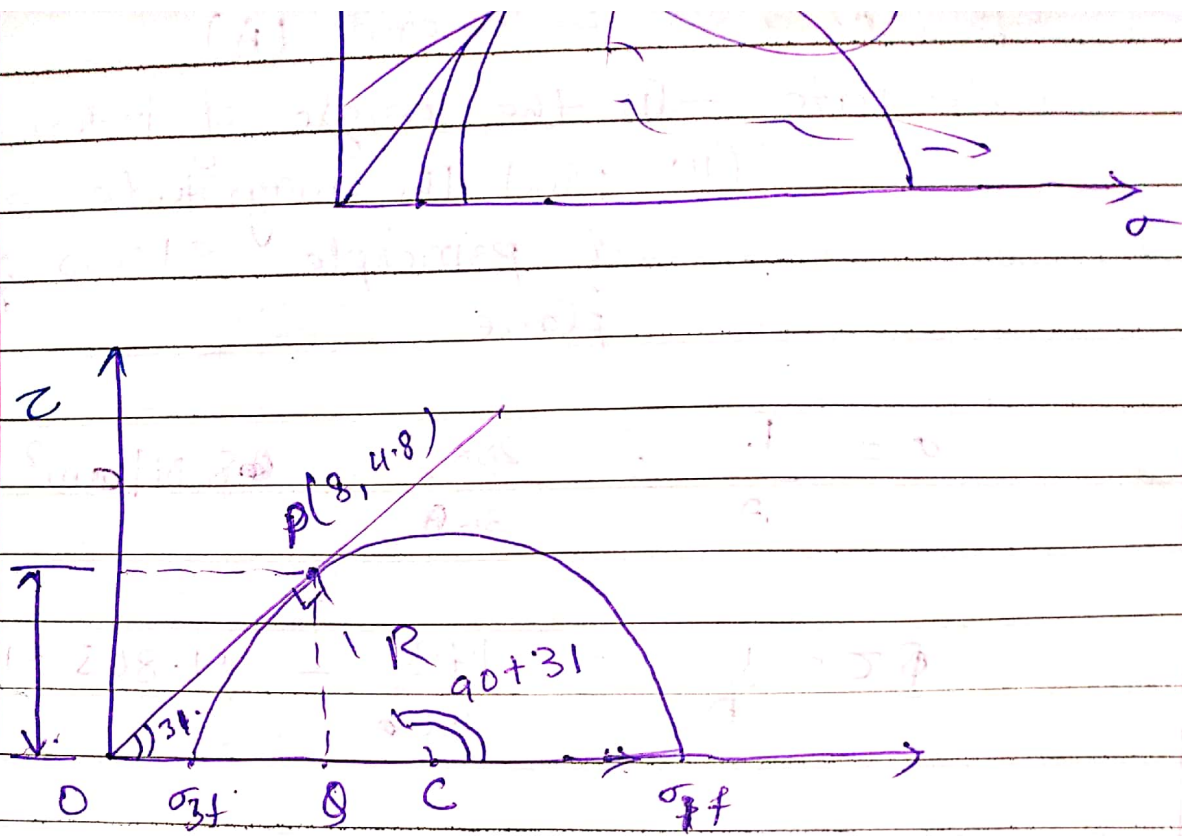
$$\frac{4.805}{8} = \tan \phi$$

~~$$\phi = \tan^{-1} \left(\frac{4.805}{8} \right)$$~~

$$\phi = \tan^{-1} \left(\frac{4.805}{8} \right)$$

$$= 30.99$$

$$\approx 31$$



gn ΔOPQ

$$OP = \sqrt{OQ^2 + QP^2}$$

$$= \sqrt{8^2 + 4.8^2}$$

$$OP = 9.33$$

gn ΔOPC

$$\tan \phi = \frac{PC}{OP}$$

$$PC = OP \tan \phi$$

$$= 9.33 \tan(30.99) = 5.60$$



$$\cos \phi = \frac{OP}{OC}$$

$$OC = \frac{OP}{\cos \phi} = \frac{9.33}{\cos(30.99)} = 10.88$$

In Mohr's circle = 121
then
in actual = $\frac{121}{2} = 60.5$

$$\sigma_{if} = C + R \Rightarrow 10.88 + 9.33 = 20.21$$

$$\sigma_{3f} = C - R \Rightarrow 10.88 - 9.33 = 1.55$$

② Triaxial test \rightarrow

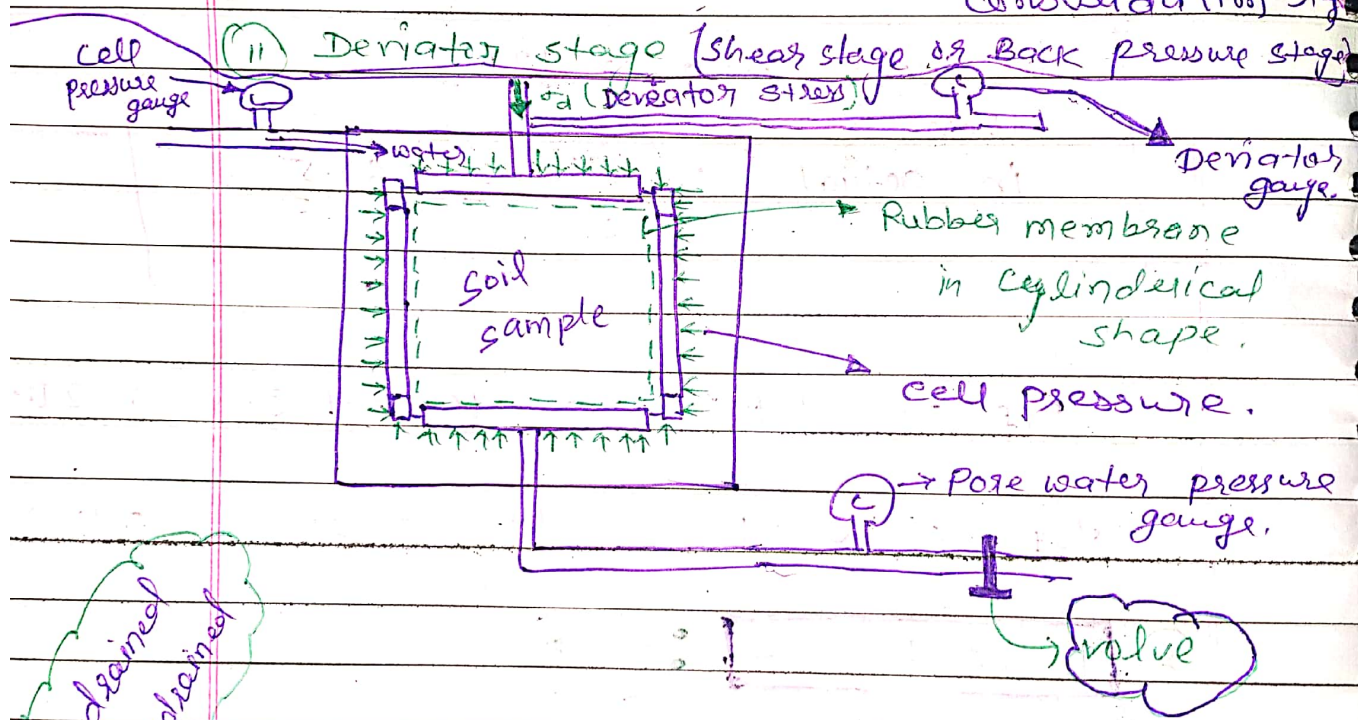
- \rightarrow This is the most commonly used shear strength test
- \rightarrow It is suitable for all types of soil.
- \rightarrow Drainage can be controlled whatever be type of soil
(i.e. sand can be tested in undrain condition and clay can be tested in drain condition.)

- excess pore water pressure generated can be measured
- volume changes can also be measured
- failure plane is not forced

Triaxial test is completed in two stages

(i) Cell pressure stage (confining stage or consolidation stage)

(ii) Deviator stage (shear stage or back pressure stage)



open → drained
close → undrained

→ Triaxial cell is filled with water and the soil specimen is sealed inside the rubber membrane and then cell pressure is applied.

→ This test is conducted in two stages in which drainage valve will be closed or open as our desire.

① Cell pressure Stage →

→ All ground stress called confining pressure or cell pressure is applied using external water pressure

→ If test is unconsolidated the drainage valve will be closed.

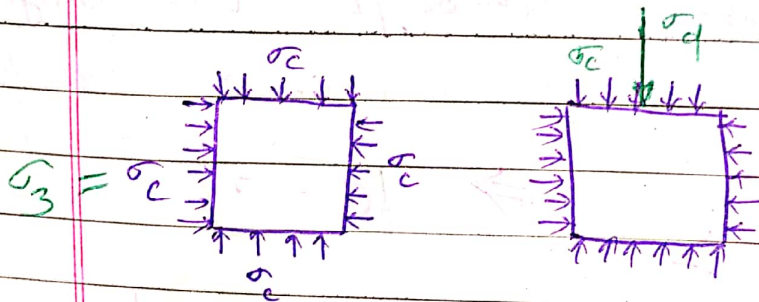
But if test is consolidated the drainage valve will be open and expulsion of pore water pressure is permitted

→ When expulsion pore water is stop, 1st stage will be completed (soil sample is not dry)

② Deviator Stage →

→ Confining pressure is kept constant and additional axial stress is applied known as deviator stage

deviator stress increase gradually until the soil fail in shear

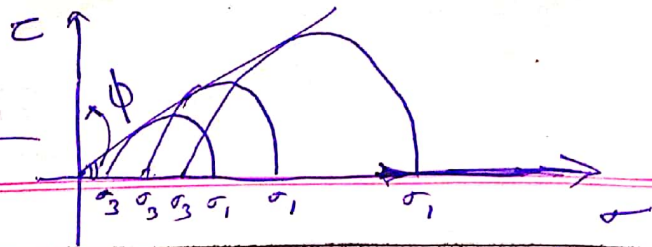


$$\sigma_1 = \sigma_c + \sigma_d$$

$$\sigma_1 = \sigma_3 + \sigma_d$$

$$\sigma_d = \sigma_1 - \sigma_3$$

At the time of failure,



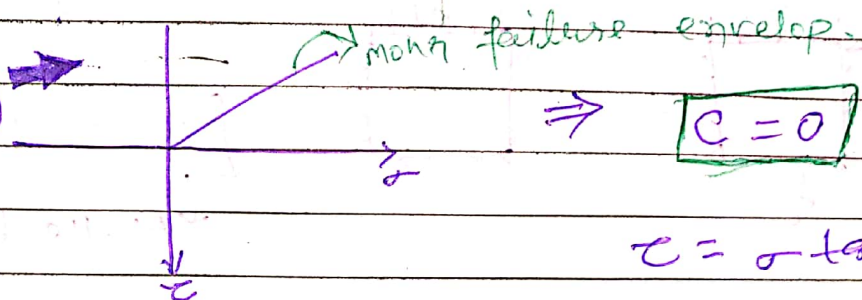
Types of triaxial test

① Consolidated drain test : → Slow test

- Drainage is allowed in both the stages, Hence this test takes longest time
- that why this is also called slow test.
- The confining pressure generated in 1st stage represents the horizontal confinement in field.
- Loading rate is so slow, so as to allow the water is expelled out.
- Pore water pressure does not buildup any stage
- The Shear strength parameters are obtained are corresponding to effective stress or total stress.

① * If the soil normally consolidated Mohr's envelop passes through the origin

By the experiment

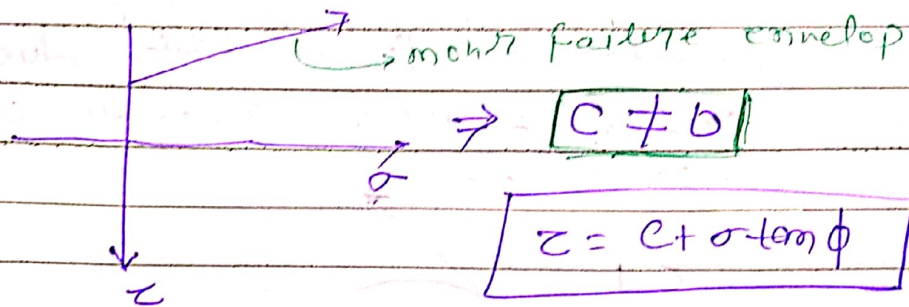


⇒ $C = 0$

$\tau = \sigma \tan \phi$

$= \bar{\sigma} \tan \phi'$

b) * If the soil over consolidated some shear strength due to preconsolidation exists and hence even at zero normal stress, it will have some shear strength.



APPLICATION: →

① This test is suitable coarse grained soil.

② consolidated undrain test : →

→ 1st stage takes around 24 hrs and 2nd stage takes 2 hrs approximately

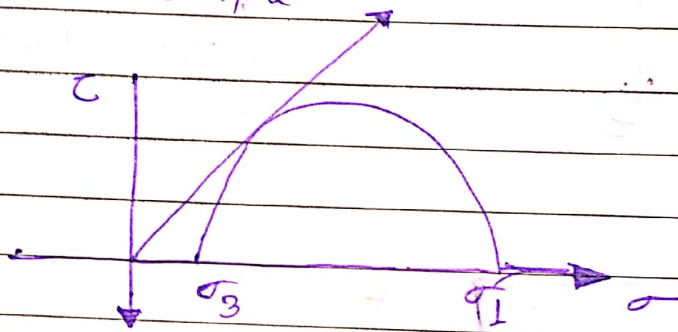
→ In this test we find out total stress parameters however if pore water pressure is measured, we can also calculate effective stress parameters also. (c' & ϕ')

If we express effective stress parameters the test symbol is denoted by CU

→ The strength of soil under CU testing is primarily governed by confining stress at the consolidation has been done in 1st stage.

* No strength will be developed in the 2nd stage.

→ In the CU testing as the confinement is increased progressively larger Mohr circle is obtained because ~~initial~~ ^{larger} due to initial confinement more strength comes into the sample



* → for normally consolidated soil $C_u = 0$

$C_{cu} = 0$
Cohesive strength of soil in consolidated undrain test

$C_{cu} \neq 0$ → for over consolidated soil.

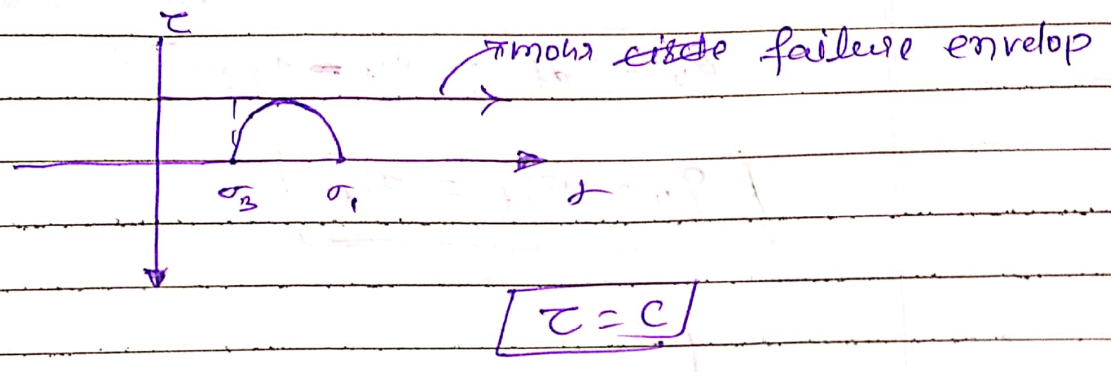
③ Unconsolidated undrain test: - (Quick test)

Consolidation is not permitted in both the stages.

Therefore it take 15 minutes only. So this called quick test.

→ we generally perform total stress analysis in this case to compare the field situation in which excess pore water pressure can not be measured.

→ strain does not build up in both the stages that why mohr failure envelop becomes horizontal.



→ This test is suitable for fine grained soil, which have low permeability

→ Loading rate is very fast.

④ Unconfined compression test : →

- No confinement is applied i.e $\sigma_3 = 0$
- Hence only one mohr circle is obtained.
- It is special case of triaxial test in which confining pressure is zero. It means there is no 1st stage (cell pressure stage)

→ Therefore no Rubber membrane is required. Without Rubber membrane dry soil and sand can not be held in position

→ Hence this test can be conducted in saturated silt and clay ~~but at~~

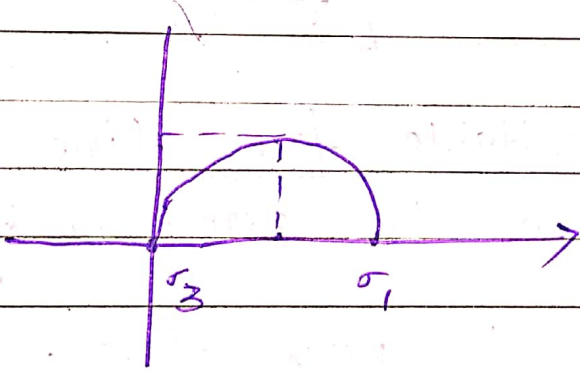
→ But it is more suitable for clay

→ $\sigma_d = \sigma_1 - \sigma_3$

at failure (σ_d is also known as unconfined compressive strength q_u)

$q_u = \sigma_{df} = \sigma_{1f} - \sigma_{3f} \rightarrow 0$

$q_u = \sigma_{df} = \sigma_{1f}$



$C = \frac{\sigma_{1f}}{2} = \frac{\sigma_{df}}{2} = \frac{q_u}{2}$

$C = \frac{q_u}{2}$

Q. The undrain cohesion of remoulded clay soil is 10 kN/m^2 . if the sensitivity of clay is 20 corresponding remoulding compressive strength is

- (a) 5 kN/m^2 (b) 10 kN/m^2
 (c) 20 kN/m^2 (d) 200 kN/m^2

$$SF = \frac{c_u(\text{undisturbed})}{c_u(\text{remoulded})}$$

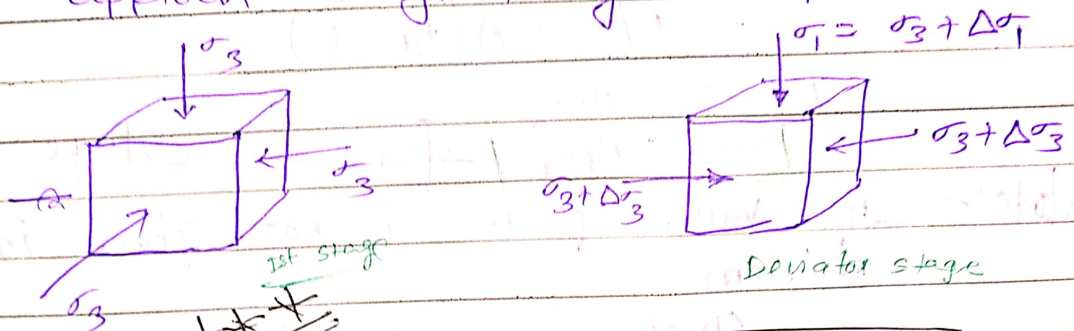
$$q_u = 2c$$

$$q_u = 2 \times 10 = 20$$

* Skempton Pore Pressure Co-efficient :-

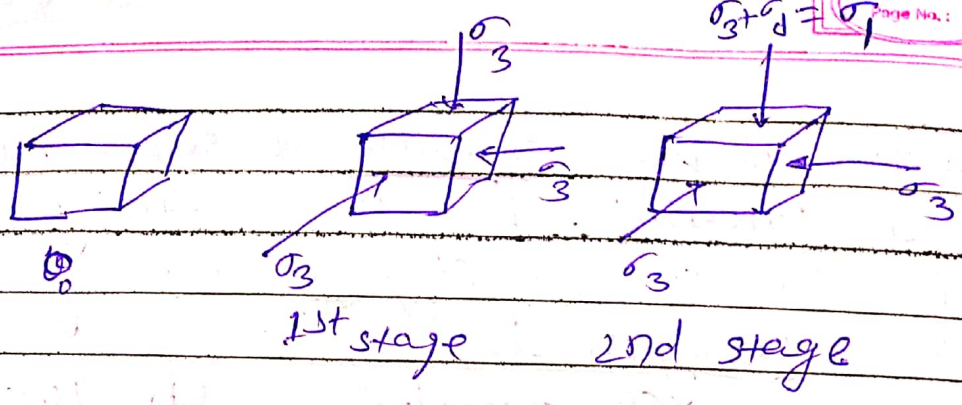
→ Pore pressure co-efficient are used to express response of pore water pressure to change in total stress in undrain condition.

→ It is not possible to measure pore pressure by practical, then theoretical approach is given by Skempton.



$$\Delta u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)]$$

A & B are pore pressure Co-efficient



	U_0	U_1	U_2
1		$\Delta\sigma_1 = \sigma_3$ $\Delta\sigma_3 = \sigma_3$	$\Delta\sigma_1 = \sigma_d$ $\Delta\sigma_3 = 0$

$$\Delta u = B \left[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3) \right]$$

$$= B \left[\sigma_3 + A(\sigma_3 - \sigma_3) \right]$$

$$\Delta u = B\sigma_3$$

$$\frac{\Delta u}{\sigma_3} = B \rightarrow \text{By 1st stage}$$

$$\Delta u = B \left[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3) \right]$$

$$= B \left[0 + A(\sigma_d - 0) \right]$$

$$\Delta u = AB\sigma_d$$

$$\Delta u = A\sigma_d \rightarrow \text{from 2nd stage}$$

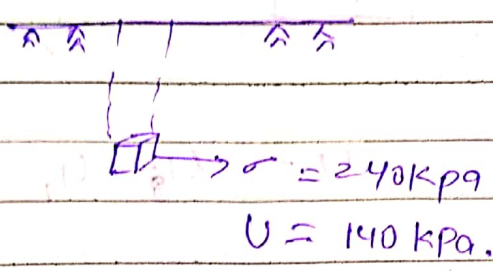
Notes- If value of B is not given in problem

① for saturated soil $B = 1$

② for dry soil $B = 0$

$$B = 0$$

Q-1 A saturated clay sample was obtained from field without allowing the water content and void ratio to change. The sample was subjected to following stress in the field.



Q.1) On sampling ~~is~~ what are the stress acting on the sample immediately.

$$\sigma = \sigma - u$$

$$= 240 - 140$$

$$= 100 \text{ kPa}$$

total stress $\sigma = 0$
effective stress $\bar{\sigma} = 100 \text{ kPa}$ (effective does not change immediately even load is applied or removed)

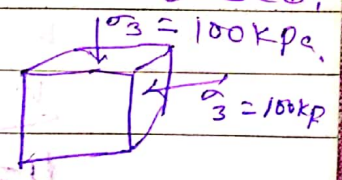
$$u = \bar{\sigma} + u$$

$$u = -\bar{\sigma} + \sigma$$

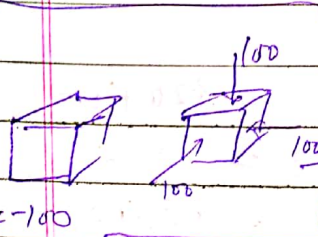
$$= -100 + 0$$

poor water pressure (u) = -100 kPa

2. Q → sample was placed on triaxial cell and cell pressure was applied at 100 kPa while keeping the drainage valve closed, what are stress on it



$$\bar{\sigma} = (\sigma - u) \Rightarrow (100 - 0)$$



$$\bar{\sigma} = 100 \text{ kPa}$$

$$\sigma = 100 \text{ kPa}$$

$$u = 0$$

$$\Delta u = u_1 - u_0$$

$$100 = u_1 - 100$$

$$u_1 = 100$$

$$\Delta u = B (\Delta \sigma_3 + A (\Delta \sigma_1 - \Delta \sigma_3)) \Rightarrow \Delta u = 1 (100 + 1/3 (100 - 100))$$

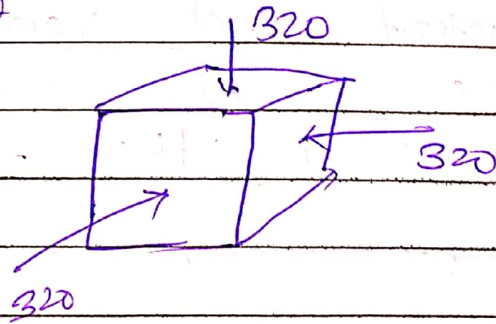
$$\Delta u = 100 \text{ kPa}$$

Q.3 Cell pressure is now raise to 320 KPa with drainage valve closed what stress on it

$$\sigma = 320 \text{ KPa}$$

$$u = 220 \text{ KPa}$$

$$\bar{\sigma} = 100 \text{ KPa}$$



$$u = u_2 - u_1$$

$$\Delta u = B(\Delta \sigma_3 + A(\Delta \sigma_1 - \Delta \sigma_3))$$

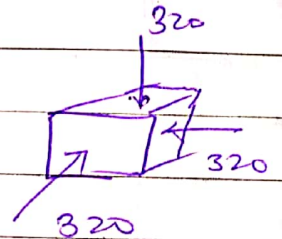
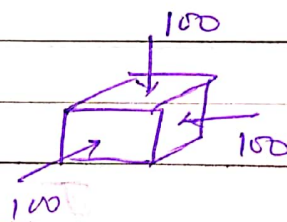
$$\Delta u = 1 [220 + A(220 - 320)]$$

$$\Delta u = 220$$

$$u_2 - u_1 = 220$$

0

$$u = 220$$

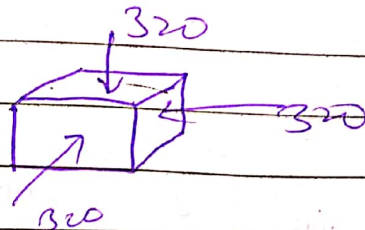


Q.4 → Drainage is now open and soil is allowed to consolidate under soil cell pressure. Find the stress.

Soln: →

valve open

$u = 0$ due to open valve
Excess pore water pressure becomes zero



$$\sigma = 320 \text{ KPa}$$

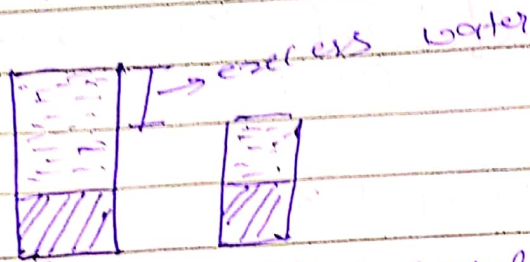
$$u_2 = 220 \text{ KPa}$$

$$\bar{\sigma} = 100 \text{ KPa}$$

$$\sigma = 320 \text{ kPa}$$

$$\bar{\sigma} = 320 \text{ kPa}$$

Q5) What will be the nature of change in water content and degree of saturation after the consolidation.

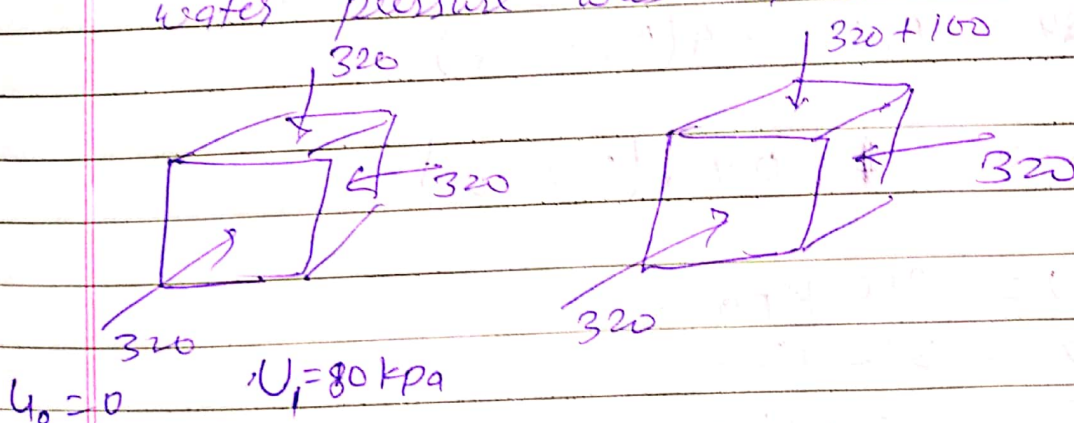


After consolidation

$$S = 100\% \text{ (Does not change)}$$

Water content and void ratio are decreases.
But degree of saturation remain constant.

Q6) Drainage valve is now closed, and addition axial pressure is applied with cell pressure remaining constant at 320 kPa. When axial pressure was 100 kPa, the pore water pressure was measure as 80 kPa.



$$\Delta u = B(\Delta \sigma_3 + A(\Delta \sigma_1 - \Delta \sigma_3))$$

$$80 - 0 = 1(\cancel{320} + A(100 - 0)) \Rightarrow \frac{80}{100} = A$$

$$\boxed{A = 0.8}$$

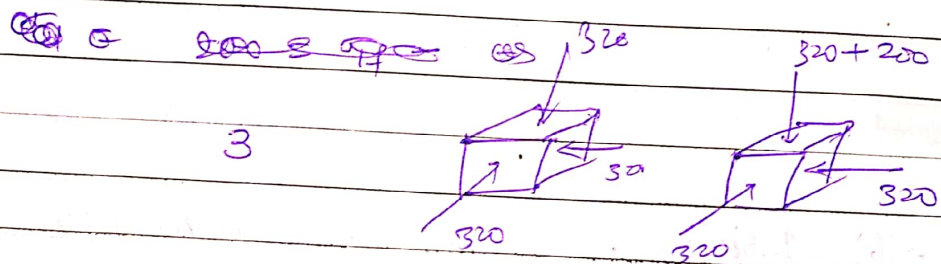
(7) \Rightarrow Had an axial stress of 200 kPa being achieved the sample has failed

(a) if $A = 1$ then what is the value $\bar{\sigma}_{1f}$

(b) find the value of ϕ' if $c = 0$

$$\sigma_1 = 200 \text{ kPa} + 320$$

$$\sigma_d = \sigma_{1f} - \sigma_{3f}$$



$$U = 0$$

$$PWP = 0$$

$$\sigma = 320$$

$$\Delta \sigma_1 = 200$$

$$\Delta \sigma_3 = 0$$

$$\Delta u = B(\Delta \sigma_3 + A(\Delta \sigma_1 - \Delta \sigma_3))$$

$$U = 0 = B(0 + 1(200 - 0))$$

$$U = 200 \text{ kPa}$$

$$\sigma = 320 \text{ kPa}$$

$$\bar{\sigma}_{1f} = 320 \text{ kPa}$$

$$\bar{\sigma}_{3f} = 320 - 200 = 120 \text{ kPa}$$

(b)

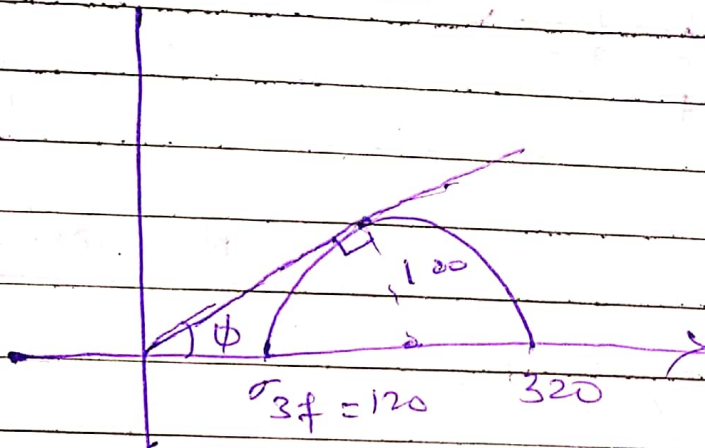
$$\tau = \sigma \tan \phi$$

$$\sigma_{1f} = \sigma_{3f} \tan^2 \left(45 + \frac{\phi}{2} \right) + 2c \tan \left(45 + \frac{\phi}{2} \right)$$

$$320 = 120 \tan^2 \left(45 + \frac{\phi}{2} \right)$$

$$\frac{320}{120} = \tan^2 \left(45 + \frac{\phi}{2} \right)$$

$$\phi = 27.03$$

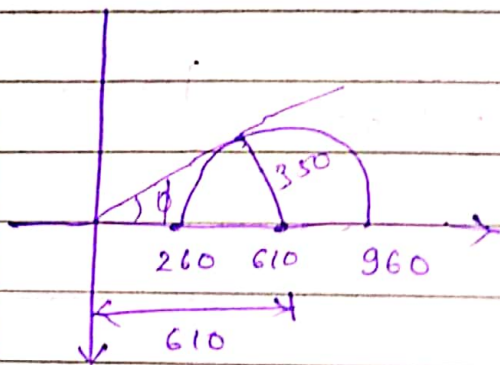


$$C = \frac{120 + 320}{2} = 250$$

$$R = \frac{320 - 120}{2} = \frac{200}{2} = 100$$

$$\sin \phi = \frac{100}{250}$$

Q → A sample of dry sand are tested in triaxial and direct shear test. In the triaxial test the sample failed when the major and minor principal stress were 960 kPa and 260 kPa, respectively. What shear strain could be expected in the direct shear stress if normal stress of 230 kPa.



$$C = \frac{960 + 260}{2}$$

$$= \frac{1220}{2} = 610$$

$$R = \frac{960 - 260}{2}$$

$$= \frac{700}{2} = 350$$

$$\sin \phi = \frac{350}{610}$$

$$\phi = \sin^{-1}\left(\frac{35}{61}\right)$$

$$= 35.01$$

In shear box test ($\sigma = 230$)

$$\tau = \sigma \tan \phi$$

$$= 230 \tan(35.01)$$

$$= 161.10 \text{ kPa}$$

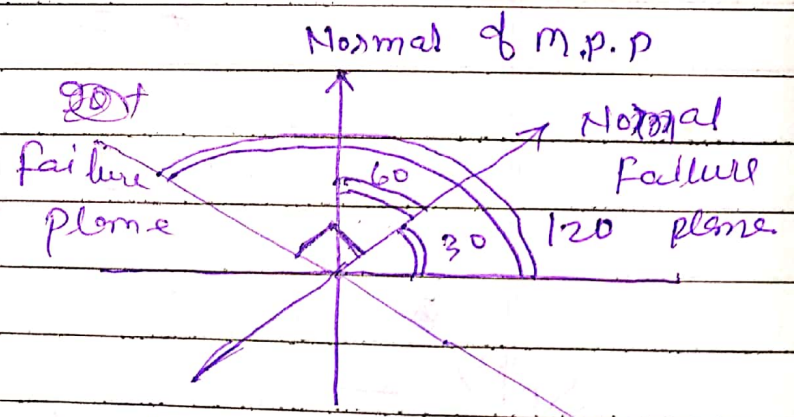
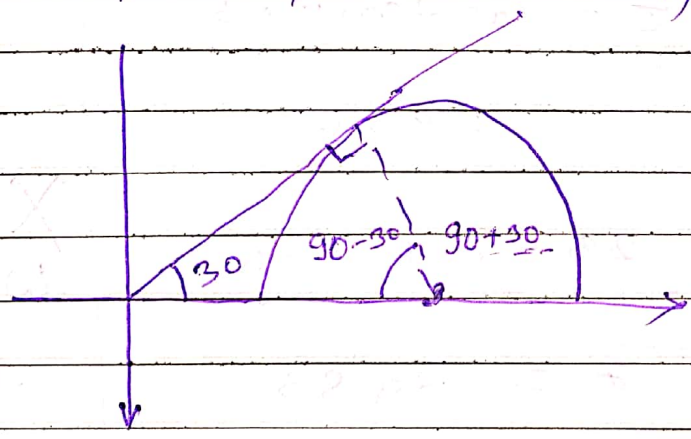
2nd method

$$\sigma_{1f} = \sigma_{3f} \tan^2 \left(45 - \frac{\phi}{2} \right) + 2c \tan \left(45 + \frac{\phi}{2} \right)$$

$c = 0$ (\because sand)

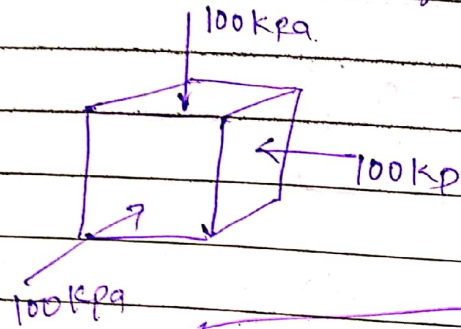
$$\sigma_{1f} = \sigma_{3f} \tan^2 \left(45 - \frac{\phi}{2} \right)$$

Q → Ques If angle of shear resistance is 30° . What will be the angle between the normal of major principle plane and failure plane.



in Mohr circle = 60°
actual in = 30°

Q. A CU test was conducted on a soil sample with cell pressure $\sigma_c = 100 \text{ kPa}$ if $c = 0$ and $\phi = 13.3^\circ$ and $c' = 0$ then $\phi' = 30^\circ$. What was the pore water pressure at failure?



$$\sigma = 100 \text{ kPa}$$

$$\tau = 100 \tan 13.3$$

$$= 23.63$$

$$23 = \bar{\sigma} \tan 30$$

$$\bar{\sigma} = 39.83$$

$$U = 60.17$$

X
Wrong
approach

Solⁿ 1 -

$$\sigma_{1f} = \sigma_{3f} \tan^2 \left(45 + \frac{\phi}{2} \right)$$

$$\sigma_{3f} = 100 \text{ kPa}$$

$$c = 0 \quad \phi = 13.3$$

$$\sigma_{1f} = 159.74 \text{ kPa}$$

Initial suction → pore water pressure in negative

let $pwp = U$

$$\bar{\sigma}_{1f} = \sigma_{1f} - U$$

$$\bar{\sigma}_{1f} = 159.74 - U \quad \text{--- (I)}$$

$$\bar{\sigma}_{3f} = 100 - U \quad \text{--- (II)}$$

$$\bar{\sigma}_{1f} = \bar{\sigma}_{3f} \tan^2 \left(45 + \frac{\phi'}{2} \right)$$

$$c = 0, \quad \phi = 30$$

$$159.74 - U = (100 - U) \tan^2 \left(\frac{45 + 30}{2} \right)$$

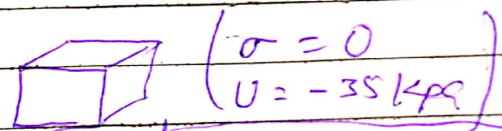
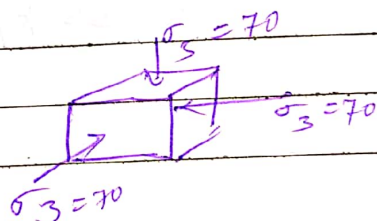
$$159.74 - U = 300 - 3U$$

$$2U = 300 - 159.74$$

$$U = 70.125 \text{ kPa}$$

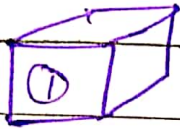
Q → A triaxial test specimen from a saturated clay sample is subjected to cell pressure is 70 kPa, initial suction in the specimen 35 kPa. If $c' = 0$ and $\phi' = 20^\circ$ and a constant $A_f = -0.2$ then determine the shear strength of soil.

$$\sigma_3 = 70 \text{ kPa}, \quad \phi' = 20^\circ, \quad c' = 0, \quad A_f = -0.2$$



Initial Suction = -ve

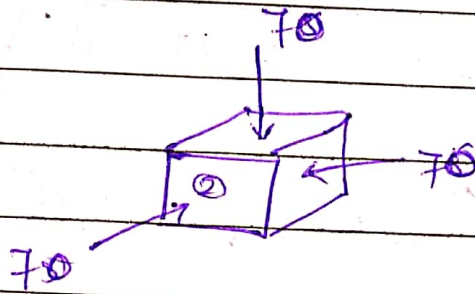
$$\Delta U = B (\Delta \sigma_3 + A / \Delta \sigma_1 - \Delta \sigma_3) \Rightarrow \dots$$



$$\sigma = 0$$

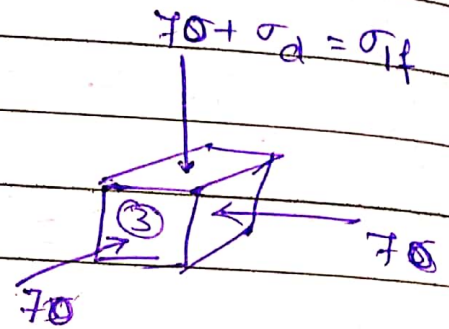
$$U_1 = -35 \text{ kPa}$$

Suction $\rightarrow (-v_g)$



Cell pressure
stage

$$U_2$$



Perforated stage.

from ① and ② system.

$$\Delta\sigma_1 = 70$$

$$\Delta\sigma_3 = 70$$

$$\Delta U = B \left(\Delta\sigma_3 + A(\Delta\sigma_1 + \Delta\sigma_3) \right)$$

$$\Delta U = 1 \left(70 + A \cancel{70} - 70 \right)$$

$$U_2 = U_1 = 70 \Rightarrow U_2 - (-35) = 70$$

~~$$U_2 = 70$$~~

$$U_2 = 70 - 35 = 35 \text{ kPa.}$$

again using relation and
from (I) and (II) stage

$$\Delta \sigma_1 = \sigma_d$$
$$\Delta \sigma_3 = 0$$

$$\Delta u = B(\Delta \sigma_3 + A(\Delta \sigma_1 - \Delta \sigma_3))$$

$$\Delta u = AB\sigma_d$$

$$u_3 - u_2 = A\sigma_d$$

$$u_3 - 35 = -0.2\sigma_d$$

from (III) anal (1)

$$\Delta \sigma_1 = 70 + \sigma_d$$

$$\Delta \sigma_3 = 70$$

$$u_3 = ?$$

$$u_1 = -35$$

$$u_3 - 35 = 1[70 + A(70 + \sigma_d - 70)]$$

$$u_3 - 35 = 70 + A\sigma_d$$

$$u_3 = 35 + 0.2\sigma_d$$

* Using relation.

$$\sigma_{1f} = \sigma_{1f} - U_3$$

$$= 70 + \sigma_d - 35 + 0.2\sigma_d$$

$$= 35 + 1.2\sigma_d$$

$$\sigma_{3f} = \sigma_{3f} - U_3$$

$$= 70 - 35 + 0.2\sigma_d$$

$$= 35 + 0.2\sigma_d$$

Again using relation.

$$\sigma_{1f} = \sigma_{3f} + \tan^2 \left(45 - \frac{\phi'}{2} \right)$$

$$35 + 1.2\sigma_d = (35 + 0.2\sigma_d) + \tan^2 \left(45 + \frac{30}{2} \right)$$

~~35 + 1.2\sigma_d = 35 + 0.2\sigma_d + \tan^2 55~~

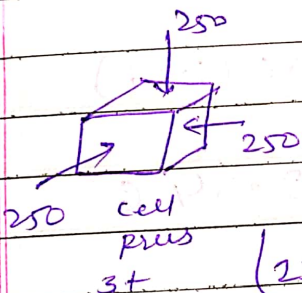
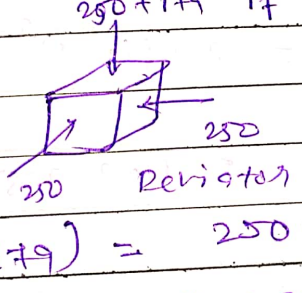
$$\tan^2 55 = 2.039$$

~~35 + 1.2\sigma_d = 35 + 0.2\sigma_d + 2.039~~

d Consolidated undrain torsional test where performed on two identical specimen of saturated clay with pore pressure measurement the observations are as follow.

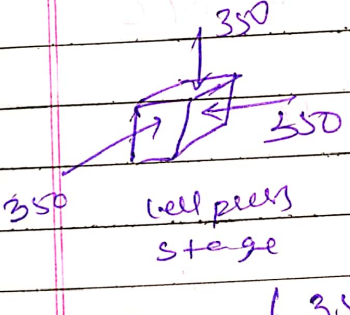
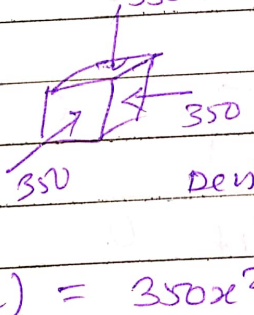
Sample	cell pressure at failure	deviator stress at failure	pore water pressure at failure
①	250 Kpa	179	101 Kpa
②	350 Kpa	242 Kpa	145 Kpa

determine the shear strength parameter in terms of total stress as well as effective stress

$$\sigma_1 = \sigma_3 + \tau \tan \phi + \frac{2c}{\sigma_3 + \tau \tan \phi}$$

$$(250 + 179) = 250 \alpha^2 + 2c\alpha \quad \text{--- (1)}$$

$$(350 + 242) = 350 \alpha^2 + 2c\alpha \quad \text{--- (2)}$$

from ① & ②

$$429 = 250 \alpha^2 + 2c\alpha$$

$$592 = 350 \alpha^2 + 2c\alpha$$

$$\frac{163}{100} = 100 \alpha^2$$

$$\alpha = \frac{163}{100} = 1.63$$

$$-100 \left(45 + \frac{\phi}{2} \right) = 1.27$$

$$45 + \frac{\phi}{2} = -100 = (1.27)$$

$$\phi = \frac{(51.78 + 45)}{2} \times 2$$

$$= \cancel{48.78} \quad 13.56$$

$$f_{1000} \quad E_2 - (1)$$

$$429 = 250x^2 + 2cx$$

$$429 = 250(1.27)^2 + 2xc(1.27)$$

$$429 = 403.225 + 2.54c$$

$$429 - 403.225 = 2.54c$$

$$\boxed{\text{Sample - 1}} \quad c = 10.14$$

$$\bar{\sigma}_{1f} = \sigma_{1f} - U$$

$$= 429 - 101$$

$$= 328$$

$$\bar{\sigma}_{3f} = \sigma_{3f} - U$$

$$= 592 - 101$$

$$= 491$$

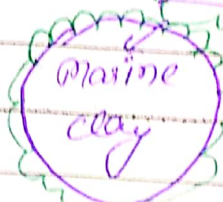
Vane Shear Test :->

-> This test is used in lab as well in field.

-> The mechanism of this test is same for lab and field.

-> In this test there is no mechanical major pore pressure and drainage facility is provided. Hence it can be conducted only undrain condition.

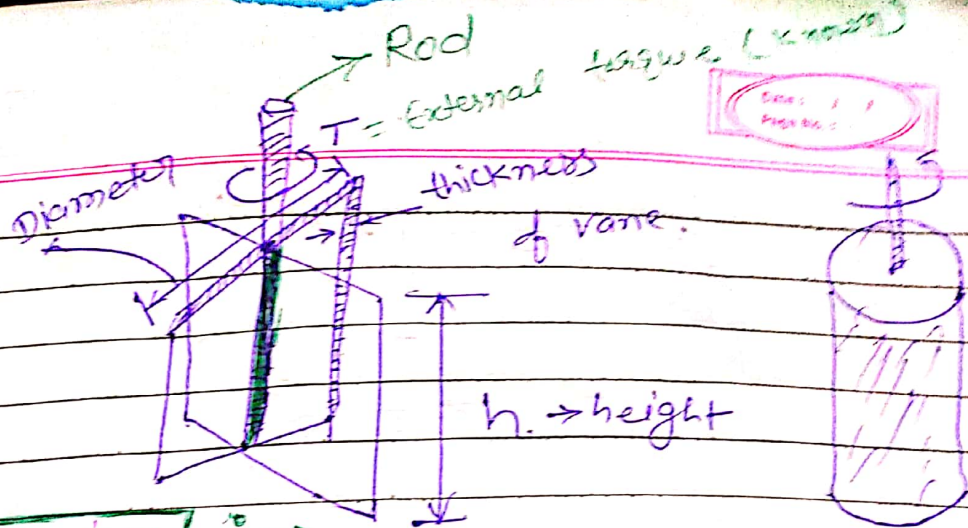
-> It is suitable for soft saturated clay



-> To find undrain shear strength and also ~~find~~ we used to find out sensitivity of clay

SIZE OF VANE SHEAR INSTRUMENT

	Lab	Field.
Height ->	20mm	10 to 20 cm
Diameter ->	12mm	5 to 10 cm.
thickness of vane ->	0.5 to 1mm	2 to 3mm.



Mechanism

→ The vane is punched into the soil and torque is applied by rotating the vane 6°/minutes.

$T_{external} = T_{side} + T_{bottom} + T_{top}$.

Condⁿ

① If vane completely inserted in soil.

$$T_{external} = T_{side} + 2 T_{bottom}$$

② If vane is inserted ground level.

$$T_{external} = T_{side} + T_{bottom}$$

* $T_{side} = (\tau \times \pi d L) \times \frac{d}{2}$
 $= \frac{\tau \pi d^2 L}{2}$

* T_{bottom}
 or top $= (\tau \cdot dA) \cdot r$
 $= \int_0^{d/2} \tau \times (2\pi r \cdot dr) \cdot r$
 $= \tau \times 2\pi \int_0^{d/2} r^2 dr$



$$= 2\pi z \times \left[\frac{r^3}{3} \right]_0^{d/2}$$

$$= 2\pi z \cdot \frac{d^3}{24}$$

$$= \frac{z \times \pi d^3}{12}$$

$$T_{\text{external}} = T_{\text{side}} + 2 T_{\text{bottom}}$$

$$= \frac{z \times \pi d^2 l}{2} + 2 \times \frac{z \times \pi d^3}{12}$$

$$T_{\text{external}} = z \left[\frac{\pi d^2 l}{2} + \frac{\pi d^3}{6} \right]$$

$$\tau = \frac{T_{\text{external}}}{\frac{\pi d^2 l}{2} + \frac{\pi d^3}{6}}$$

(2) When a vane is inserted ground level.

$$\tau = \frac{T_{\text{external}}}{\frac{\pi d^2 l}{2} + \frac{\pi d^3}{12}}$$